FORBIDDEN FREQUENCIES CRITERION

1. INTRODUCTION

The analysis of structures sometimes requires the avoidance of some frequencies in order to avoid resonance problems.

When the exciting forces are rotatory an exciting frequency can be defined (typically related with the engine frequency and associated with a SEI: Sustained Engine Imbalance caused by a fan blade-off event for example) that creates a dynamic excitation on the structures. If the natural frequency of the structure under analysis is coincident with the exciting frequency the static response is multiplied by a factor much higher than 1, meaning that a design based on static loads will lead to a premature failure of the structure.

In order to avoid this effect is quite usual to define frequencies that have to be avoided. This frequency range is typically part of the specification of the structures as a requirement to be fulfilled. For example: "The structure has to be designed with a minimum natural frequency of 30Hz".

This method is useful to understand how this frequency is defined and how to use the correct mode to check the requirement.

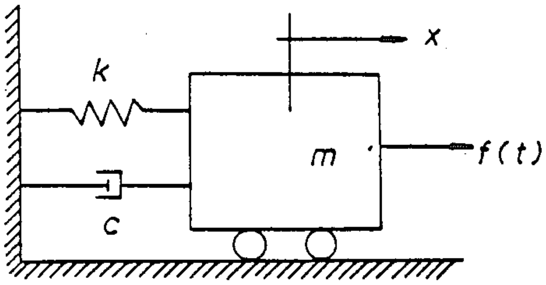
2. THEORETICAL BACKGROUND

The basic definitions to be taken into account are:

- Exciting frequency (ω_e): It is the frequency of the dynamic force that can be considered the external force. It is typically due to the engine of the aircraft.
- Natural frequency (ω_n): It is the frequency of the structural system in absence of damping that exactly compensates the inertial forces with the elastic forces without external forces. It is calculated as the solution of the following equation:

$$-\omega_n{}^2[M]\{x\}+[K]\{x\}=\{0\}$$

- SDOF: Single Degree of freedom system. It is a basic system comprised of a mass, a spring element, and a damper with only one degree of freedom. This system is typically used to explain the behavior of more complex systems (with several degrees of freedom) because each vibration mode behaves like a SDOF with some specific masses, stiffness and damping (especially when the damping is proportional)



- Critical damping: It is the value of the damping c that changes the nature of the solution of the equation from underdamped to overdamped:

$$c_{crit} = 2m\omega_n$$

- Critical damping ratio: It is the ratio (normally in percentage) of the real damping and the critical damping. The usual values for typical structures are between 0.02 and 0.05.

$$\xi = c/c_{crit}$$

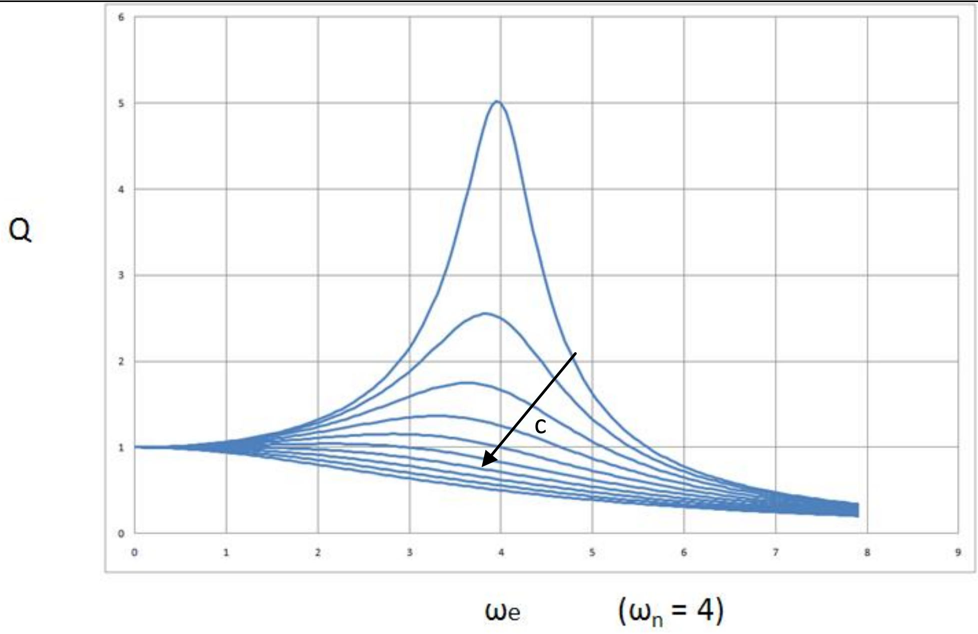
- Amplification factor (Q): The amplification factor is the dynamic response of a system at a frequency divided by the static response.

$$Static\ Response = \{x\}\ From\ the\ solution\ of\ [K]\{x\} = \{f\}$$

$$Dynamic\ Response = \{x\}\ From\ the\ solution\ of\ [M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{f\}$$

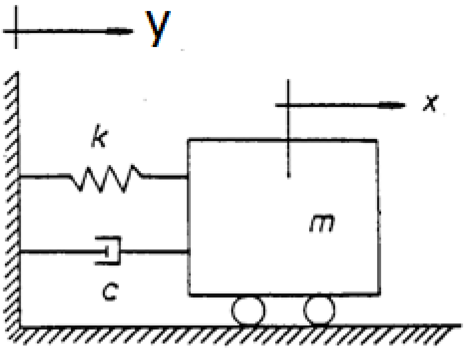
$$Q = \frac{1}{\sqrt{\left(1 - \frac{{\omega_e}^2}{{\omega_n}^2}\right)^2 + \left(2\xi\frac{\omega_e}{\omega_n}\right)^2}}$$

The amplification factor follows the next graph according to the ratio of the excitation and natural frequency and the value of the critical damping ratio.



It can be noted that the maximum of the amplification factor (resonance) is not exactly at the natural frequency but at a value slightly lower. As the damping increases, the maximum of the amplification factor is at a larger distance from the natural frequency.

- Enforced Vibrations on a SDOF: The enforced vibrations on a SDOF are the vibrations induced by the movement of the support instead of the action of a external force as on the SDOF system:

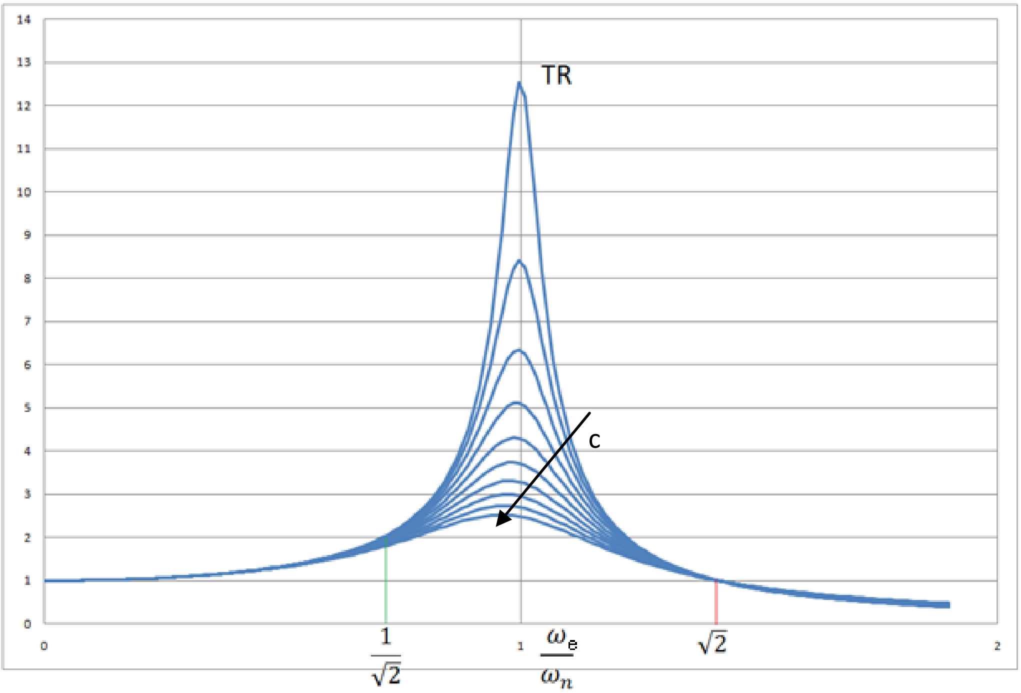


The vibrations on the structures are normally induced by the movement of the support and not by a direct dynamic force. Therefore, this system is the most common case on the stress analysis of structures.

- Transmissibility (TR): The transmissibility is a concept similar to the amplification factor applied to the enforced vibrations on a SDOF. It is defined as the module of the movement of the mass divided by the module of the movement of the support in enforced vibrations.

$$TR = \frac{|x|}{|y|} = \frac{\sqrt{1 + \left(2\xi \frac{\omega_e}{\omega_n}\right)^2}}{\sqrt{\left(1 - \frac{{\omega_e}^2}{{\omega_n}^2}\right)^2 + \left(2\xi \frac{\omega_e}{\omega_n}\right)^2}}$$

The transmissibility follows the next graph according to the ratio of the excitation and natural frequency and the value of the critical damping ratio:



It can be noted that the maximum of the transmissibility is not exactly at the natural frequency but at a value slightly lower. As the damping increases, the maximum of the transmissibility is at a larger distance from the natural frequency.

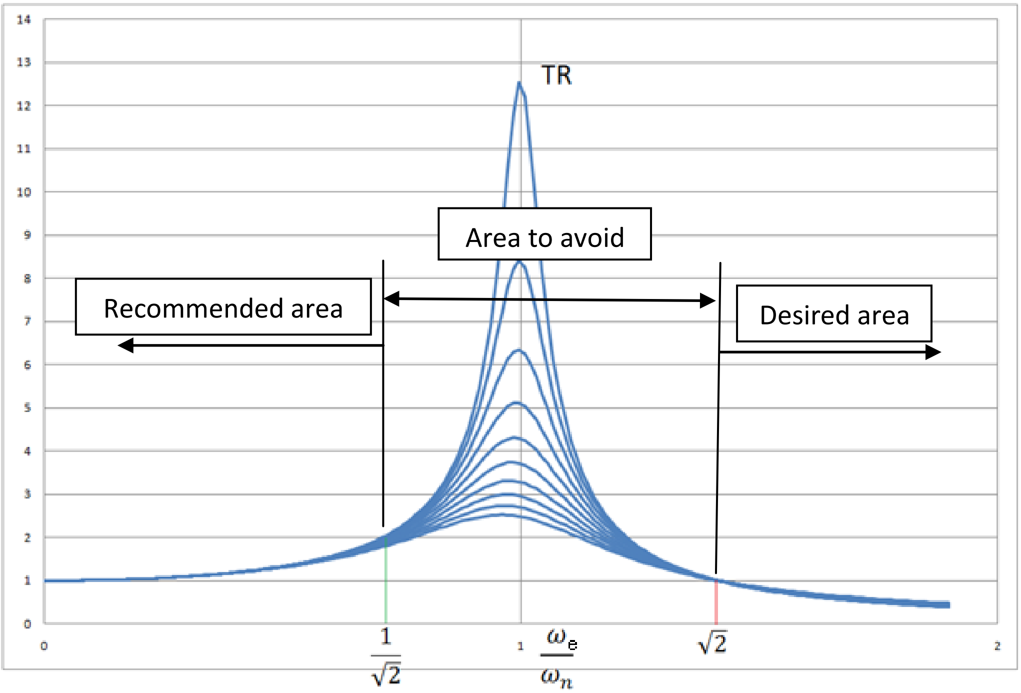
- Effective Mass of vibration: The effective masses of vibration are the contributions of each mode to the total mass of the system on a specific direction. On the system, the total mass can be different on each direction. Each vibration mode has a contribution to the total mass on each direction depending on how it vibrates. Adding all the contributions (effective masses) for each mode, the total mass on the direction investigated is got. This concept is applicable to multiple degrees of freedom systems.

In different words, the effective mass of a mode ‘i’ on a direction ‘r’ indicates how much mass moves this mode ‘i’ on this direction ‘r’. It is a quantitative way to determine the importance of a mode on the global behavior of the structure (under the dynamic point of view). Normally the lowest modes concentrate most of the total mass.

3. CRITERION FOR FORBIDDEN FREQUENCIES

The most usual scenario when performing dynamic analyses of structures is to deal with vibrations induced by the supporting structure. Normally, the stress engineer has to perform a dynamic analysis whose excitation comes from the points where the bracket or system is attached to. Therefore, the most useful concept is the transmissibility and the effective masses.

Taking a look to the transmissibility graph, three important zones can be differenced:

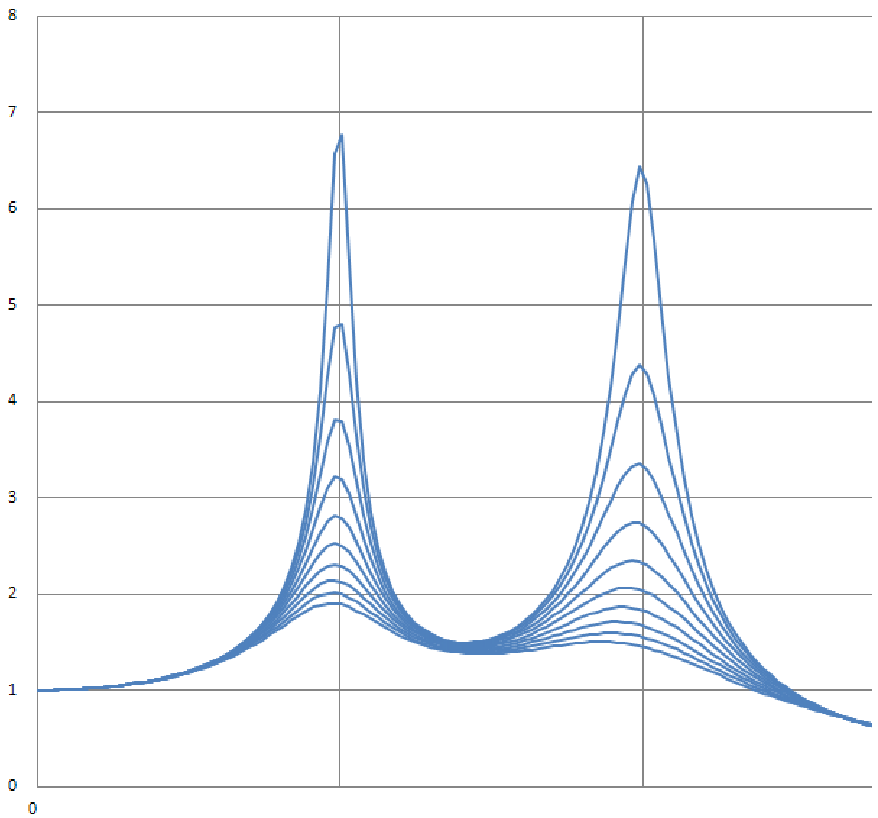


- Area to avoid: The area situated between $1/\sqrt{2}$ and $\sqrt{2}$ is the area where the transmissibility is highest with any damping ratio. Therefore, it is the worst possible area to be used for the design of the structure.

- Desired Area: This area ensures transmissibility lower than 1 for any damping ratio. Therefore, it is the perfect area to be used for designing the structure. If the engineer can change his structure in order to get a natural frequency ω_n such that $\omega_n < \omega_e/\sqrt{2}$ then he is ensuring that the excitation movement of the support will not create significant vibrations on the structure (vibrations not higher than the static response calculated with the amplitude of the dynamic movement).

In different words, if the excitation frequency is high enough (higher than $\sqrt{2}\omega_n$) the dynamic effect is lower than the static effect with the same amplitude. Therefore, the high excitation frequencies are not impacting on the design.

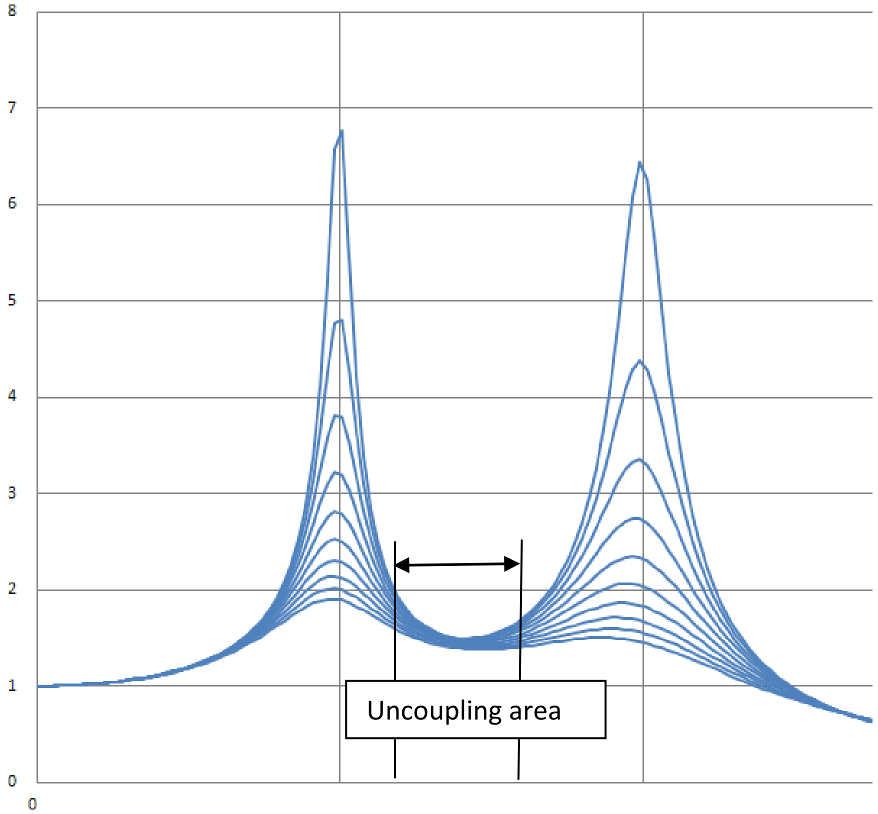
This situation is normally impossible to be attained because the structural systems are not SDOF but MDOF (multiple degrees of freedom). They have a lot of possible vibration modes and it is practically impossible to ensure that all of them with a significant effective mass associated will fulfill that they are lower than $\omega_e/\sqrt{2}$. In practice, the only possibility is that the excitation frequency is really high, which is not the case normally.



- Recommended area: This area ensures that the transmissibility of the system is lower than 2 for any damping ratio. Therefore, if the engineer can change his structure in order to get a natural frequency ω_n such that $\omega_n > \omega_e \cdot \sqrt{2}$ then he is ensuring that the excitation movement of the support will not create significant vibrations on the structure (vibrations not higher than twice the static response calculated with the amplitude of the dynamic movement).

This area is recommended and typically used for the design of the structures because it is enough to ensure that the minimum frequency of the structure (the first vibration mode) is higher than $\omega_e \cdot \sqrt{2}$ to have a clear limitation on the dynamic effects of the excitation. If this requirement is fulfilled, the dynamic response can be estimated as twice (conservative assumption) the static response with the same amplitude of movement of the support, and the design can be calculated with a static analysis (NASTRAN SOL 101) loading the structure with twice the amplitude of the vibration. If a less conservative assumption is required the real transmissibility can be calculated with a conservative value of the damping ratio in order to reduce the value of 2. For this purpose the SDOF formula can be used as the range between the minimum frequency of the system and 0 Hz is not coupled with any other vibration mode. However, if higher accuracy is required a frequency response analysis can be performed by FEM (NASTRAN SOL 111).

Sometimes, there are several frequencies to be checked (or it is just impossible to get a first natural frequency so high as the requirement suggests) and just an uncoupling criterion can be stated. Typically for MDOF systems the uncoupling criterion is to be on the range between both peaks:



In this uncoupling area:

$$\omega_n^{i+1} > \sqrt{2}\omega_e$$

$$\omega_n^i < \frac{\omega_e}{\sqrt{2}}$$

Where the index 'i' indicates the number of vibration mode.

In order to evaluate if the frequencies fulfill the criterion, only the frequencies with significant effective masses should be taken into account. The frequencies with low values of effective mass will be related with local vibration modes that are not relevant in the design of the whole structure. An exception for this would be the local vibration modes that could potentially lead to the loss of functionality of a system. In order to request the effective masses on NASTRAN (from version 2001) the following case control statement is recommended:

MEFFMASS(PRINT, MEFFM, FRACSUM)=YES

The output of this command on the f06 file looks like this:

MODAL EFFECTIVE MASS FRACTION
(FOR TRANSLATIONAL DEGREES OF FREEDOM)

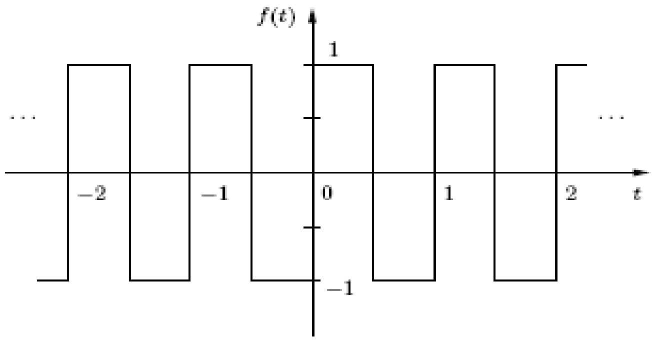
MODE NO.	FREQUENCY	T1 FRACTION	T1 SUM	T2 FRACTION	T2 SUM	T3 FRACTION	T3 SUM
1	1.809512E-03	3.231795E-01	3.231795E-01	1.187544E-03	1.187544E-03	7.019066E-02	7.019066E-02
2	1.421094E-03	4.383788E-01	7.615583E-01	2.399939E-02	2.518694E-02	4.550195E-01	5.252102E-01
3	8.568414E-04	1.468586E-02	7.762442E-01	6.705720E-01	6.957589E-01	1.135627E-02	5.365664E-01
4	5.582778E-04	9.161858E-02	8.678627E-01	4.015950E-03	6.997749E-01	3.195549E-01	8.561214E-01
5	2.226882E-03	1.320593E-01	9.999220E-01	4.559656E-04	7.002309E-01	1.438725E-01	9.999939E-01
6	4.326689E-03	7.796379E-05	1.000000E+00	2.997691E-01	1.000000E+00	6.165209E-06	1.000000E+00
7	1.681558E+02	3.728804E-22	1.000000E+00	4.239179E-24	1.000000E+00	7.618804E-21	1.000000E+00
8	3.225124E+02	8.234257E-24	1.000000E+00	1.997833E-23	1.000000E+00	3.030938E-23	1.000000E+00
9	3.750325E+02	4.556185E-25	1.000000E+00	1.753200E-23	1.000000E+00	3.725049E-23	1.000000E+00
10	4.181057E+02	5.960653E-24	1.000000E+00	1.403739E-24	1.000000E+00	6.895718E-23	1.000000E+00
11	5.073476E+02	2.964566E-24	1.000000E+00	1.233461E-25	1.000000E+00	3.939806E-26	1.000000E+00
12	5.234438E+02	1.506805E-24	1.000000E+00	2.272347E-25	1.000000E+00	5.419749E-23	1.000000E+00
13	5.853464E+02	6.372085E-23	1.000000E+00	5.978718E-25	1.000000E+00	2.445567E-25	1.000000E+00
14	6.331703E+02	2.574230E-23	1.000000E+00	1.528974E-24	1.000000E+00	4.692681E-23	1.000000E+00
15	8.160795E+02	7.780237E-25	1.000000E+00	1.718920E-25	1.000000E+00	2.831016E-24	1.000000E+00

There are two columns for each of the main directions (T1, T2, T3, R1, R2, and R3). The output shown is the MASS FRACTION which is the most relevant parameter. From these columns it can be figured out that the first mode moves in T1 the 32.318% of the total mass of the FEM in T1 direction (there is a mass associated with each direction). The second mode moves in T1 the 43.838% of the mass of the FEM in T1, and the first five modes concentrate practically the 100% of the mass in T1 direction (see SUM column, which is a cumulative mass counting). Similar conclusions can be extracted for the T2 and T3 directions (or R1 to R3 if they were shown). Therefore, the relevant modes on this case are the first five modes, as the highest modes hardly have any mass associated.

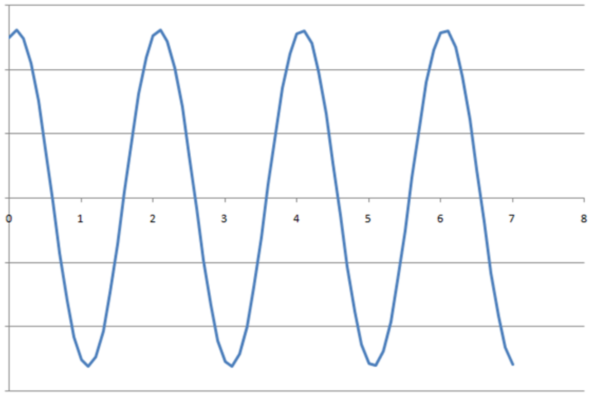
4. CONSIDERATIONS ABOUT EXCITATION FREQUENCY

The excitation is normally periodic (there are exceptions such as random analyses which are managed in a different way) because they come from rotatory elements (such as engines) but this fact does not mean that they are harmonic.

A periodic excitation means that the excitation signal follows a periodic pattern:



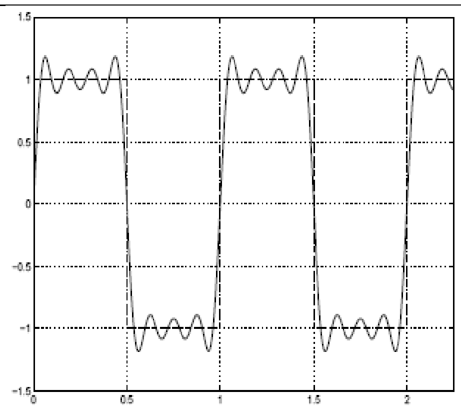
An harmonic excitation means that the excitation follows a sinusoidal pattern:



The Fourier transform shows that any periodic function can be written as a sum of harmonic functions, as an example for the function above:

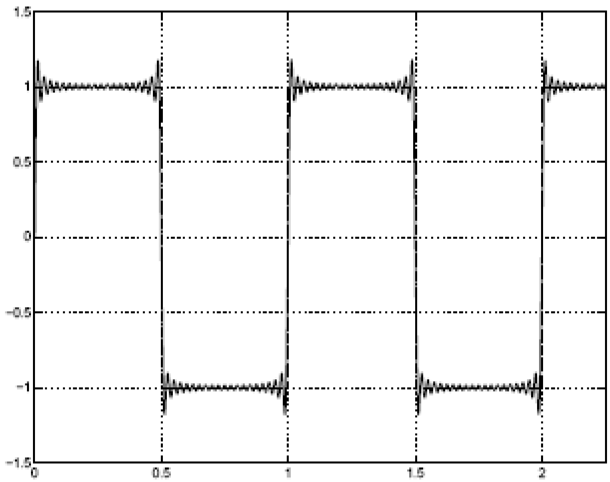
$$f(t) = \frac{4}{\pi}\sum_{k=0}^{\infty}\frac{1}{2k+1}\sin 2\pi(2k+1)t$$

This sum is more similar to the real function as the number of terms is increased:



(9 terms)

First term = 0 sin(2πt) (mean value of the signal)
Second term = 0.4244 sin(6πt)
Third term = 0.2546 sin(10πt)
Fourth term = 0.1819 sin(14πt)
…..



(39 terms)

As the excitation signal is normally periodic, it is really a sum of harmonic signals with a frequency multiple of the periodic signal. These terms are commonly called harmonics of the signal and their importance is lower as the order on the term is higher.

Therefore, when there is a periodic excitation, it is not only important to take into account the basic frequency of the signal (ω_e=2π or f=1 Hz on the example) but also the multiples of the frequency because they create the harmonics. It is true that the importance of the harmonics decay quickly as the amplitude is significantly reduced as the order of the harmonic increases, but the first ones can be relevant on the periodic signal, and hence on the response of the structure.

Thus, when a periodic signal is known the harmonics have to be calculated in order to evaluate when they can be neglected according to the relevance amplitude of the harmonic. If the harmonics are not taken into account when an excitation frequency is provided, there is a risk to get a resonance on the design. Normally, the first five harmonics are considered. Consequently the criterion of forbidden frequency would be that the first natural frequency has to be higher than the frequency of the fifth harmonic of the excitation multiplied by $\sqrt{2}$. However, the relative importance of the higher harmonics depends on the shape of the excitation, if the excitation signal looks like a sinusoidal is clear that the relative importance of the harmonics will be practically negligible, just the contrary would happen if the signal is very different to a harmonic signal.