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HOW RBE1 RBE2 AND RBE3 WORK?

1. INTRODUCTION

The RBE are a kind of MPC that are commonly used on the NASTRAN models to simulate parts whose stiffness is unknown or when the loading has to be implemented on the FEM and the distribution is not clear.

The following document is an explanation of how the different RBE work and the implications on the model of using one or another.

2. RBE1/RBE2

The RBE1 and RBE2 are both infinitely stiff MPC. This means that the different degrees of freedom connected by RBE1 or RBE2 will behave as if they were part of an infinitely stiff item. This does not mean that the different degrees of freedom will have the same displacements (translations and rotations). Only if the nodes connected are coincident the same displacements will be obtained on both nodes for any loading.

The RBE2 is an MPC were one single node is defined as independent (providing its six translations as independent degrees of freedom) and any degrees of freedom of other nodes are defined as dependent.

The dependent degrees of freedom of the RBE2 are calculated following the equation of a rigid freebody:

$$\{T\}_{dependent} = \{T\}_{independent} + \{R\}_{independent} \times \{\overrightarrow{ID}\}$$

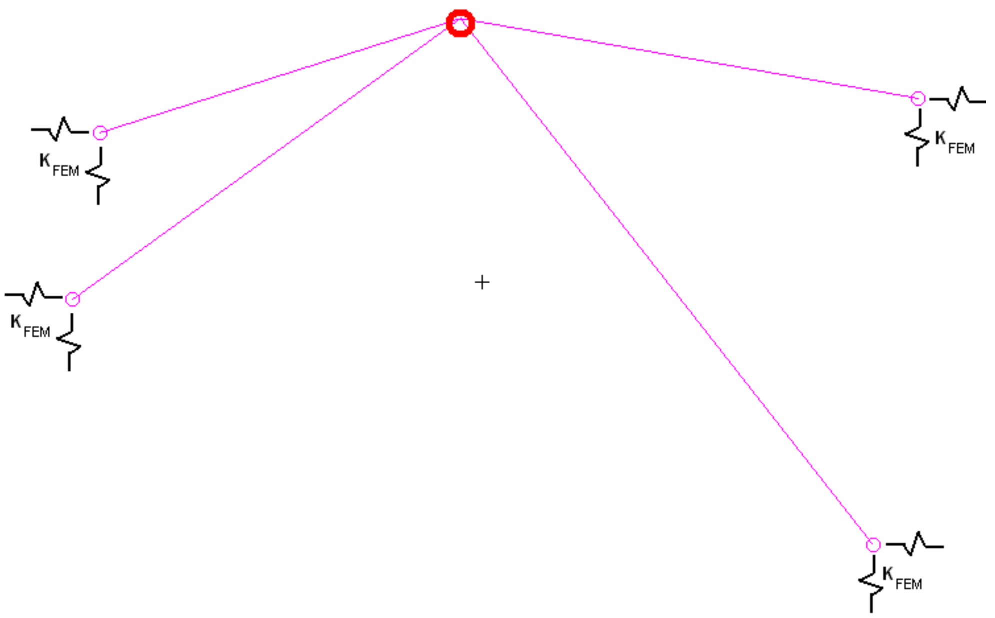
$$\{R\}_{dependent} = \{R\}_{independent}$$

Where:

{T}: Translations vector = {T1,T2,T3}
{R}: Rotations vector={R1,R2,R3}
{ID}: Vector from the independent (base) to the dependent node (tip)={X_d-X_i,Y_d-Y_i,Z_d-Z_i}
X: Vectorial product

Once the geometry is set on the model (and therefore the {ID} vectors are known), the values of the dependent degrees of freedom only depend on the values of the independent degrees of freedom.

The RBE2 is normally attached in the following way:



The central node (in red) is the independent one whereas the other 4 nodes are the dependent ones (in magenta)

The dependent nodes are usually connected to some nodes of the FEM and therefore have a stiffness associated for each degree of freedom.

If a displacement is enforced on the independent node of the RBE2, the dependent degrees of freedom will be calculated according to the equations above. In this case, the reactions of the structure (and MPCFORCES at the dependent nodes) will depend on the stiffness of the FEM associated with each degree of freedom but the displacements obtained on the dependent degrees of freedom only depend on the displacements on the independent node. Therefore, the reactions on the MPC depend on the model stiffness but the displacements do not. The SPCFORCE on the independent node will be calculated as the total sum of the MPCFORCES coming from each dependent degree of freedom (the moment calculation will also take into account the forces and arms). Obviously the SPCFORCE on each node will also depend on the FEM stiffness.

If a load is applied on the independent node of the RBE2, the dependent degrees of freedom will be calculated according to the equations above. But the value of the independent degrees of freedom is unknown, and therefore, the solution is not directly attainable. In this case, the equations can be identified with the displacement multiplied by the "condensed" stiffness of the FEM on each direction on this dependent degrees of freedom [K_{FEM}]:

$$\{T\}_{dependent} = \{T\}_{independent} + \{R\}_{independent} \times \{\overrightarrow{ID}\} = [K_{FEM}]\{MPCFORCE\}_{dependent}$$

$$\{R\}_{dependent} = \{R\}_{independent} = [K_{FEM}]\{MPCFORCE\}_{dependent}$$

Where:
[K_{FEM}]: It is the "condensed" stiffness matrix of the FEM on the dependent degrees of freedom analyzed
$\{MPCFORCE\}_{dependent}$: MPCFORCE on the dependent degrees of freedom analyzed

At the end, there are so many equations as unknowns plus 6. The unknowns are the MPCFORCE on each dependent degree of freedom plus the 6 displacements (3 translations and 3 rotations) of the independent node. In order to complete the 6 remaining equations and make the system of equations solvable, the equilibrium has to be added (for the 3 forces and 3 moments):

$$\sum_{dependent} \{MPCFORCES\} = \{F\}_{applied}$$

With all these 6 equations the system is solvable as the applied loading is known.

The solution of the system changes with the stiffness of the FEM if a force is externally applied. It changes also depending on the dependent degrees of freedom included (123 only or 123456). Therefore, the repartition of the loading and the displacements are not easily computable by hand calculations. Anyway, some conclusions can be extracted from the explanation above:

- If the dependent degrees of freedom are connected by rotational degrees of freedom (456) the rotation of this nodes will be the same that the one of the independent node, therefore, the rotations do not depend on the position of the node, and the moments transferred on these nodes will only depend on the rotational stiffness on these nodes on the FEM.
- The displacements on the dependent degrees of freedom when the independent node has a enforced displacement applied do not depend on the FEM stiffness, only on the position of the nodes and the enforced displacements on the independent node (translations) or only on the enforced rotations on the independent (rotations)
- All the displacements of the degrees of freedom connected by the RBE2 behave

like an infinitely stiff item. Their relative distances never change, and they have a stiffening effect on the structure, not only between the independent node and the dependent ones, but also between a dependent node and another dependent node.

On the other hand, the RBE1 behaves like a RBE2; the only difference is that on the RBE1 the independent degrees of freedom can be distributed between different nodes. In this case the ID vector components are the difference between the dependent node position components and the component of the node where the corresponding rotation is assigned as independent.

3. RBE3

The RBE3 are not infinitely stiff MPC. This fact does not imply that the RBE3 do not add stiffness on the model as has been commonly said sometimes. The RBE3 can connect two parts of the FEM, making the FEM stiffer. In fact, an RBE2 connecting two coincident nodes on their six degrees of freedom and an RBE3 connecting two coincident nodes on their six degrees of freedom behave in the same way, performing an absolutely stiff connection between these two nodes.

Anyway, it is true that the connection between the independent degrees of freedom do not add additional stiffness between them.

The RBE3 calculates the displacement of the reference node as a weighted mean of the displacements of the independent nodes.

Contrary to the RBE2, where the key point is the equation relating the displacements, on the RBE3 the key point is the equation relating the loads. On the RBE3 the loads are calculated distributing the loading applied on the reference node (dependent degrees of freedom) to the independent degrees of freedom.

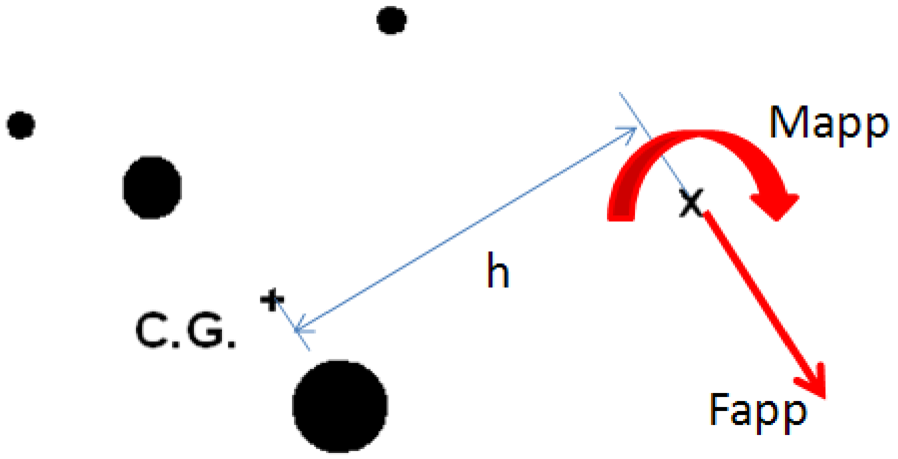
The first step is to calculate the position of the center of gravity of the independent degrees of freedom. This step is performed taking into account the weighting factor defined on the RBE3. For the independent degrees of freedom and their position:

$$\{CG\} = \frac{\sum w_i \cdot \{D\}_i}{\sum w_i}$$

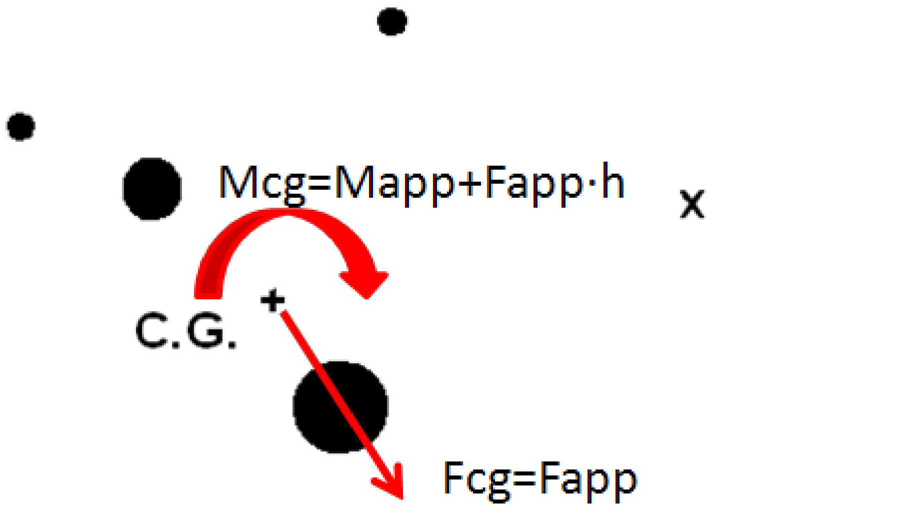
Where:
{CG}: Position of the center of gravity

w_i: Weighting factor for the degree of freedom i
$\{D\}_i$: Position of the node related with the degree of freedom i
Once the CG position is calculated, the loading is translated from the reference point where it is applied to the center of gravity by performing a equilibrium equivalency:

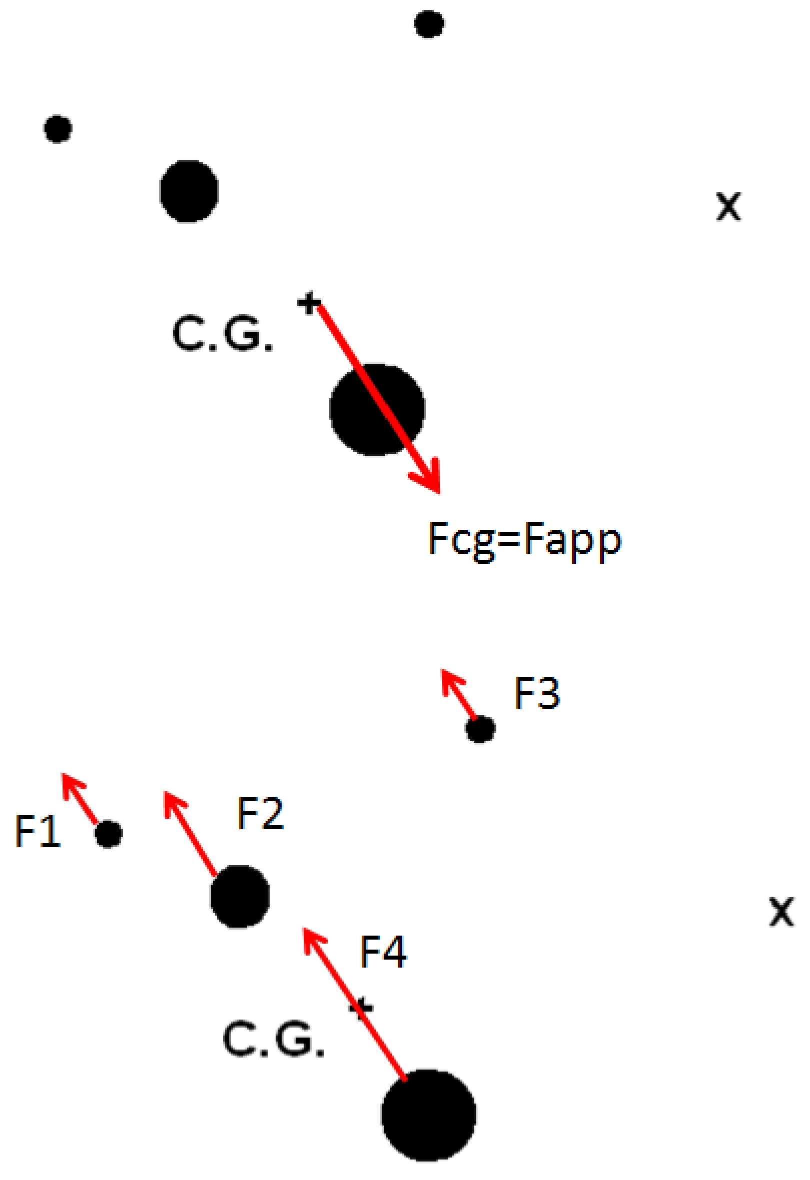
Initial Situation



Loading translated to the CG:

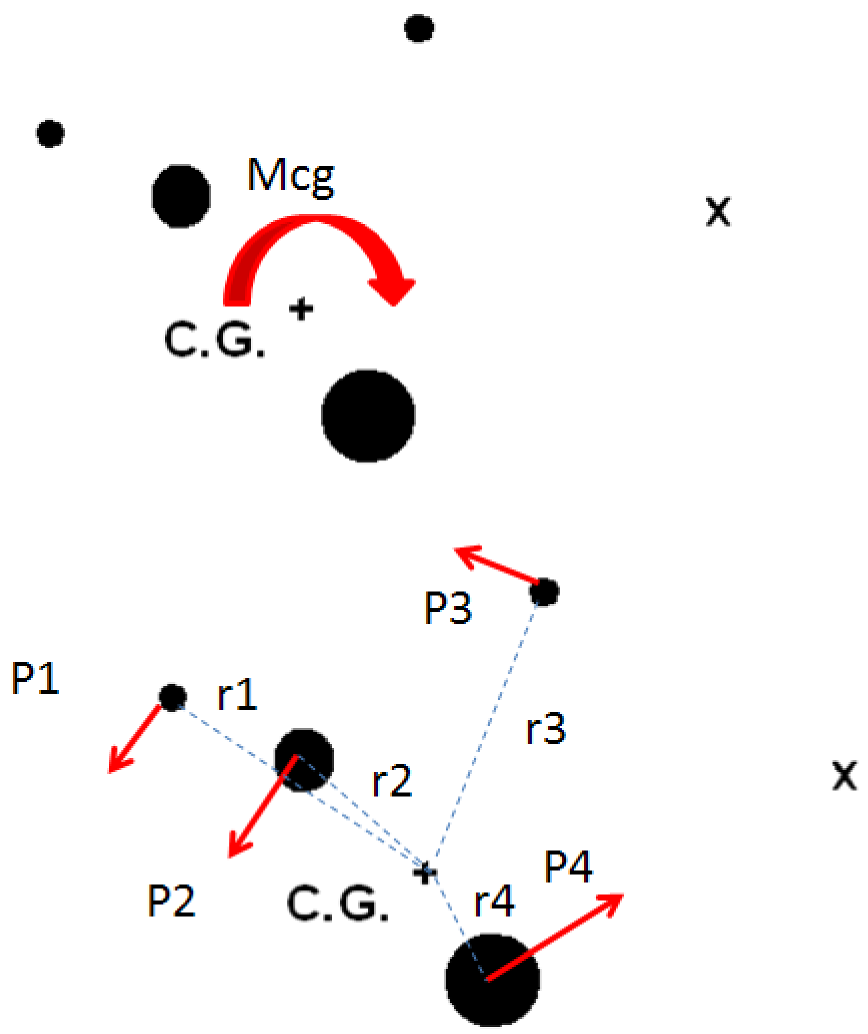


The Force Fcg is assumed to be reacted (shared) by the independent degrees of freedom according to the relative weight of each one of them. The reaction force is aligned with the Fcg force if the independent degrees of freedom allow it:



$$\{F_i\} = \{Fcg\} \cdot \frac{w_i}{\sum w_i}$$

The Moment Mcg is assumed to be reacted (shared) by the independent degrees of freedom according to the relative weight of each one of them and the relative position with regards to the CG. The reaction force that compensates the moment is in a direction perpendicular to the vector joining the CG with the position of the node where the independent degrees of freedom are selected.



$$\{P_i\} = -w_i \cdot \frac{\{Mcg\} \times \{r_i\}}{\sum w_i \cdot \{r_i\} \cdot \{r_i\}}$$

Where {ri} is the vector from the CG position to the node $\{r_i\} = \{X_i-X_{cg},Y_i-Y_{cg},Z_i-Z_{cg}\}$

The total reaction force on the independent degrees of freedom is the sum of $\{F_i\}+\{P_i\}$ on each independent degree of freedom. This force is not dependent on the stiffness of the

FEM. It is only dependent on the weighting factors and the relative position of the nodes where the dependent and independent degrees of freedom are selected.

On the other hand, as the forces are not dependent on the stiffness of the model, the displacements of the independent and dependent degrees of freedom are clearly dependent on this stiffness. Once the reaction forces are obtained on the independent degrees of freedom, the analysis of the structure is carried out according to the stiffness of the model and the displacements on the independent degrees of freedom can be obtained. The displacement on the dependent node are calculated as a mean value of the displacements on the independent nodes in the following way:

<u>RBE3 with 123 on the independent nodes and 123456 on the dependent node</u>

The following steps are performed by NASTRAN to calculate the displacements on the dependent node:

- The position of the CG is calculated as explained above
- The vectors $\{r_i\}$ are calculated as explained above
- The weighted average of the translations on the independent nodes is calculated
- An unknown rotation is assumed by NASTRAN and the displacements on the independent nodes are written as if they were calculated with a rigid body movement with the weighted average translations and the unknown rotations for each node i:
$$\{T_i\}_{independent}^{assumed} = \{T\}_{weighted\ average} + \{R\}_{unknown} \times \{r_i\}$$
- The differences between the independent displacements calculated in this way and the real ones (already known by NASTRAN as the model has been solved) is calculated as a residue for each node/component:
$$\{Residue_i\} = \{T_i\}_{independent}^{assumed} - \{T_i\}_{independent}^{FEM}$$
- The error of the unknown rotation is expressed as:
$$Error = \sum w_i \cdot \{Residue_i\} \cdot \{Residue_i\}$$
- The unknown rotation is calculated as the one that minimizes the Error using optimization algorithms
- The unknown rotation is the rotation of the dependent node
- The translations on the dependent node are calculated as:
$$\{T\}_{dependent} = \{T\}_{weighted\ average} + \{R\} \times \{r_d\}$$
Where $\{r_d\}$ is the vector from the CG to the dependent node $\{X_d\text{-}X_{cg}, Y_d\text{-}Y_{cg}, Z_d\text{-}Z_{cg}\}$

The following NASTRAN example is included to clarify the calculation process:

HOW RBE1 RBE2 AND RBE3 WORK?

```
PBUSH   1     K     1.    1.    1.    1.    1.    1.
CBUSH   1     1     11                        0
CBUSH   2     1     12                        0
CBUSH   3     1     13                        0
CBUSH   4     1     14                        0

RBE3    4           5     123456  1.    123   11    12
        13    5.    123   14

GRID    5           0.    0.    7.
GRID    11          -10.  0.    0.
GRID    12          -10.  10.   0.
GRID    13          10.   -10.  -3.
GRID    14          10.   10.   0.

FORCE   1     5     0     1.    1.    0.    0.
```

- Calculation of the CG position:

$$\{CG\} = \frac{\sum w_i \cdot \{D\}_i}{\sum w_i} = \{5 \quad 6.25 \quad -0.375\}$$

- Calculation of $\{r_i\}$:

$\{r_5\} = \{-5\ -6.25\ 7.375\}$

$\{r_{11}\} = \{-15\ -6.25\ 0.375\}$

$\{r_{12}\} = \{-15\ 3.75\ 0.375\}$

$\{r_{13}\} = \{5\ -16.25\ -2.625\}$

$\{r_{14}\} = \{5\ 3.75\ 0.375\}$

- NASTRAN can obtain the forces applied on the independent nodes through the RBE3 and solve the FEM obtaining the following displacements for nodes 11 to 14:

D I S P L A C E M E N T   V E C T O R

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
5	G	5.709755E-01	1.366698E-01	2.254698E-02	-9.085795E-03	2.261196E-02	3.164558E-03
11	G	1.731780E-01	-1.046150E-01	1.835362E-01	0.0	0.0	0.0
12	G	1.037735E-01	-1.046150E-01	1.970909E-01	0.0	0.0	0.0
13	G	2.041809E-01	3.826036E-02	-8.602905E-02	0.0	0.0	0.0
14	G	5.188676E-01	1.709697E-01	-2.945981E-01	0.0	0.0	0.0

- The weighted average of the translations is calculated as:

$$\{T\}_{weighted\ average} = \frac{\sum w_i \cdot \{T\}_i}{\sum w_i}$$

For this case the resulting value is:

$\{T\}_{\text{weighted average}}$ = { 3.844338E-01 8.548486E-02 -1.472991E-01}

- An unknown rotation is assumed by NASTRAN and the displacements on the independent nodes are written as if they were calculated with a rigid body movement with the weighted average translations and the unknown rotations for each node i:

$$\{T_i\}_{independent}^{assumed} = \{T\}_{weighted\ average} + \{R\}_{unknown} \times \{r_i\}$$

As an example, the translation TX of the node 11 is written as:

TX_{11}=3.844338E-01 + RY · 0.375 – RZ·(-6.25)

- The differences between the independent displacements calculated in this way and the real ones (already known by NASTRAN as the model has been solved) is calculated as a residue:

As an example, the residue of the translation TX of the node 11 is written as:

Residue TX_{11}=[3.844338E-01 + RY · 0.375 – RZ·(-6.25)] – [1.73178E-01]

- The error of the unknown rotation is expressed as:

$$Error = \sum w_i \cdot \{Residue_i\} \cdot \{Residue_i\}$$

- The unknown rotation is calculated as the one that minimizes the Error using optimization algorithms

The resulting rotations applying optimization is:

{R} = {-9.08580E-03 2.26120E-02 3.16456E-03}

- The resulting rotation {R} is the rotation of the dependent node

- The translations on the dependent node are calculated as:

$$\{T\}_{dependent} = \{T\}_{weighted\ average} + \{R\} \times \{r_d\}$$

Where $\{r_d\}$ is the vector from the CG to the dependent node $\{X_d\text{-}X_{cg},Y_d\text{-}Y_{cg},Z_d\text{-}Z_{cg}\}$

For the example proposed:

$\{r_d\}=\{-5 \;\; -6.25 \;\; 7.375\}$

TX_5= 3.844338E-01 + 2.26120E-02 · 7.375 – 3.16456E-03 · (-6.25) = 5.709758E-01

TY_5= 8.548486E-02 + 3.16456E-03 · (-5) – (-9.08580E-03) · (7.375) = 1.366698E-01

TZ_5= -1.472991E-01 + (-9.08580E-03) · (-6.25) – 2.26120E-02 · (-5) = 2.254715E-02

RBE3 with 123456 on the independent nodes and 123456 on the dependent node being the moment applied perpendicular to all the $\{r_i\}$ vectors and $\{r_d\}$ vector

The following steps are performed by NASTRAN to calculate the displacements on the dependent node:

- The position of the CG is calculated as explained above
- The vectors $\{r_i\}$ are calculated as explained above
- A characteristic length is calculated by NASTRAN in the following way:

$$Lc = \frac{\sum_{i=1}^{n} \sqrt{\overrightarrow{ID}_i \cdot \overrightarrow{ID}_i}}{n}$$

Where: n is the number of independent nodes

$\{ID_i\}$: Vector from the independent (base) to the dependent node (tip)=$\{X_d\text{-}X_i,Y_d\text{-}Y_i,Z_d\text{-}Z_i\}$

- The following weighting factors related with the moment (MX or MY or MZ) repartition are calculated for each node for the translational and rotational components:

$WT_i(MX)=r_{iY}^2+r_{iZ}^2$

$WT_i(MY)=r_{iX}^2+r_{iZ}^2$

$WT_i(MZ)=r_{iX}^2+r_{iY}^2$

$WR_i(MX)=Lc^2$

$WR_i(MY)=Lc^2$

$WR_i(MZ)=Lc^2$

- The percentage of moment directly absorbed by the 456 DOF of each independent node i is calculated as:

$$\%MXi = \frac{WRi(MX)}{\sum WTi(MX) + WRi(MX)}$$

$$\%MYi = \frac{WRi(MY)}{\sum WTi(MY) + WRi(MY)}$$

$$\%MZi = \frac{WRi(MZ)}{\sum WTi(MZ) + WRi(MZ)}$$

- The remaining moment is transferred to the independent nodes by means of forces following the approach explained for the RBE3.
- On this step NASTRAN has the loading on the independent nodes and can solve the model. The displacements and rotations are calculated by NASTRAN on the independent nodes. The displacements on the dependent node are calculated on the next steps.
- The weighted average of the translations on the independent nodes is calculated
- An unknown rotation is assumed by NASTRAN and the displacements on the independent nodes are written as if they were calculated with a rigid body movement with the weighted average translations and the unknown rotations for each node i:

$$\{T_i\}_{independent}^{assumed} = \{T\}_{weighted\ average} + \{R\}_{unknown} \times \{r_i\}$$

- The differences between the independent displacements calculated in this way and the real ones (already known by NASTRAN as the model has been solved) is calculated as a residue for each node/component:

$$\{Residue_i\} = \{T_i\}_{independent}^{assumed} - \{T_i\}_{independent}^{FEM}$$

- The error of the unknown rotation is expressed as:

$$Error = \sum w_i \cdot \{Residue_i\} \cdot \{Residue_i\}$$

- The unknown rotation {R}={RX,RY,RZ} is calculated as the one that minimizes the Error using optimization algorithms
- The unknown rotation is part of the rotation of the dependent node. The rotation on the dependent node is calculated as:

$$RX_{dependent} = \sum \%MXi \cdot RXi + \frac{\sum(WTi(MX) \cdot RX)}{\sum WTi(MX) + WRi(MX)}$$

$$RY_{dependent} = \sum \%MYi \cdot RYi + \frac{\sum(WTi(MY) \cdot RY)}{\sum WTi(MY) + WRi(MY)}$$

$$RZ_{dependent} = \sum \%MZi \cdot RZi + \frac{\sum(WTi(MZ) \cdot RZ)}{\sum WTi(MZ) + WRi(MZ)}$$

- The translations on the dependent node are calculated as:

$$\{T\}_{dependent} = \{T\}_{weighted\ average} + \{RX_{dependent}\ RY_{dependent}\ RZ_{dependent}\} \times \{r_d\}$$

Where $\{r_d\}$ is the vector from the CG to the dependent node $\{X_d\text{-}X_{cg}, Y_d\text{-}Y_{cg}, Z_d\text{-}Z_{cg}\}$

The following NASTRAN example is included to clarify the calculation process:

```
PBUSH   1    K    1.    1.    1.    1.    1.    1.
CBUSH   1    1    11                        0
CBUSH   2    1    12                        0
CBUSH   3    1    13                        0

RBE3    4         2     123456 1.     123456  1     3
        4
GRID    1         -10.   0.    0.
GRID    2          0.    0.    0.
GRID    3         10.    0.    0.
GRID    4          0.   10.    0.

MOMENT  1    2    0    1.    0.    0.    1.
```

- Calculation of the CG position:

$$\{CG\} = \frac{\sum w_i \cdot \{D\}_i}{\sum w_i} = \{0 \quad 3.33333 \quad 0\}$$

- Calculation of $\{r_i\}$:

$\{r_1\} = \{-10\ -3.33333\ 0\}$
$\{r_2\} = \{0\ -3.33333\ 0\}$
$\{r_3\} = \{10\ -3.33333\ 0\}$
$\{r_4\} = \{0\ 6.66666\ 0\}$

- A characteristic length is calculated by NASTRAN

$\{ID_1\}=\{-10\ 0\ 0\} \rightarrow (\{ID_1\} \cdot \{ID_1\})^{0.5} = 10$
$\{ID_3\}=\{10\ 0\ 0\} \rightarrow (\{ID_3\} \cdot \{ID_3\})^{0.5} = 10$
$\{ID_4\}=\{0\ 10\ 0\} \rightarrow (\{ID_4\} \cdot \{ID_4\})^{0.5} = 10$
$Lc= (10 + 10 +10)/3 = 10$

- The following weighting factors related with the moment (MX or MY or MZ) repartition are calculated for each node for the translational and rotational components:

$WT_i(MX)=r_{iY}^2+r_{iZ}^2$
$WT_i(MY)=r_{iX}^2+r_{iZ}^2$
$WT_i(MZ)=r_{iX}^2+r_{iY}^2$
$WR_i(MX)=Lc^2$

$WR_i(MY)=Lc^2$
$WR_i(MZ)=Lc^2$

	MX	MY	MZ
WT1	11.1111	100.0000	111.1111
WT3	11.1111	100.0000	111.1111
WT4	44.4444	0.0000	44.4444
WR1	100.0000	100.0000	100.0000
WR3	100.0000	100.0000	100.0000
WR4	100.0000	100.0000	100.0000
WT1+WT3+WT4+WR1+WR3+WR4	366.6666	500.0000	566.6666

- The percentage of moment directly absorbed by the 456 DOF of each independent node i is calculated as:

$$\%MXi = \frac{WRi(MX)}{\sum WTi(MX) + WRi(MX)}$$

$$\%MYi = \frac{WRi(MY)}{\sum WTi(MY) + WRi(MY)}$$

$$\%MZi = \frac{WRi(MZ)}{\sum WTi(MZ) + WRi(MZ)}$$

	MX	MY	MZ
%M1	0.272727	0.2000	0.1764706
%M3	0.272727	0.2000	0.1764706
%M4	0.272727	0.2000	0.1764706
%M1+%M3+%M4	0.818181	0.6000	0.5294118

- On this step NASTRAN can obtain the loading applied on the independent nodes through the RBE3 and solve the FEM obtaining the following displacements for nodes 1, 3 and 4:

D I S P L A C E M E N T   V E C T O R

POINT ID.	TYPE	T1	T2	T3	R1	R2	R3
1	G	5.882353E-03	-1.764706E-02	0.0	0.0	0.0	1.764706E-01
2	G	3.141868E-01	0.0	0.0	0.0	0.0	9.425606E-02
3	G	5.882353E-03	1.764706E-02	0.0	0.0	0.0	1.764706E-01
4	G	-1.176471E-02	0.0	0.0	0.0	0.0	1.764706E-01

Note that as the stiffness on the CBUSH is 1.0 and the external moment is unitary as well (and in Z direction), the rotations obtained on the independent nodes is coincident with the %M1, %M3 and %M4 for MZ.

- The weighted average of the translations on the independent nodes is calculated

$$\{T\}_{weighted\ average} = \frac{\sum w_i \cdot \{T\}_i}{\sum w_i}$$

For this case the resulting value is:

$\{T\}_{weighted\ average} = \{0\ \ 0\ \ 0\}$

- An unknown rotation is assumed by NASTRAN and the displacements on the independent nodes are written as if they were calculated with a rigid body movement with the weighted average translations and the unknown rotations for each node i:

$$\{T_i\}_{independent}^{assumed} = \{T\}_{weighted\ average} + \{R\}_{unknown} \times \{r_i\}$$

See an example on the paragraphs for the RBE3 (123) case

- The differences between the independent displacements calculated in this way and the real ones (already known by NASTRAN as the model has been solved) is calculated as a residue:

See an example on the paragraphs for the RBE3 (123) case

- The error of the unknown rotation is expressed as:

$$Error = \sum w_i \cdot \{Residue_i\} \cdot \{Residue_i\}$$

See an example on the paragraphs for the RBE3 (123) case

- The unknown rotation is calculated as the one that minimizes the Error using optimization algorithms

The resulting rotations applying optimization is:

$\{R\} = \{0\ \ 0\ \ 0.001764706\}$

- The rotation on the dependent node is calculated as:

$$RX_{dependent} = \sum \%MXi \cdot RXi + \frac{\sum(WTi(MX) \cdot RX)}{\sum WTi(MX) + WRi(MX)}$$

$$RY_{dependent} = \sum \%MYi \cdot RYi + \frac{\sum(WTi(MY) \cdot RY)}{\sum WTi(MY) + WRi(MY)}$$

$$RZ_{dependent} = \sum \%MZi \cdot RZi + \frac{\sum(WTi(MZ) \cdot RZ)}{\sum WTi(MZ) + WRi(MZ)}$$

$RX_{dependent}$= [(0.2727·0+0.2727·0+0.2727·0)]+[(11.111·0+11.111·0+44.444·0)]/366.6666 = 0

$RY_{dependent}$= [(0.2·0+0.2·0+0.2·0)]+[(100·0+100·0+0·0)]/500 = 0

$RZ_{dependent}$= [(0.17647·0.17647+0.17647·0.17647+0.17647·0.17647)]
+[(111.111·0.001764706+111.111·0.001764706+44.444·0.001764706)]/566.666
= 0.09425606

- The translations on the dependent node are calculated as:

$$\{T\}_{dependent} = \{T\}_{weighted\ average} + \{RX_{dependent}\ RY_{dependent}\ RZ_{dependent}\} \times \{r_d\}$$

Where $\{r_d\}$ is the vector from the CG to the dependent node $\{X_d-X_{cg}, Y_d-Y_{cg}, Z_d-Z_{cg}\}$

For the example proposed:

$\{r_d\}=\{0\ \ -3.33333\ \ 0\}$

TX_2= 0 + 0·0 – 0.09425606· (-3.3333) = 3.141869E-01
TY_2= 0 + 0.09425606·0 – 0·0 = 0
TZ_2= 0 + 0·(-3.33333) – 0·0 = 0

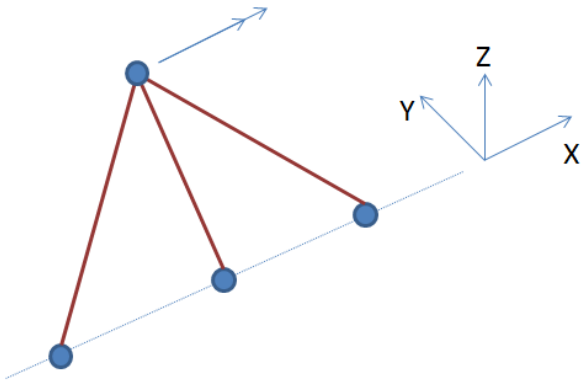
When the moment applied is not perpendicular to all the $\{r_i\}$ vectors and $\{r_d\}$ vector there are some correction factors not explained on this document.

4. ADVICE ABOUT USING RBE3

Using 456 DOF on the independent nodes of the RBE3 has to be avoided. Normally, they provide unexpected results.

The only exception is including ONE rotational DOF when the independent nodes are aligned and a hinge is created around the axis that joins the independent nodes. For this case, ONLY the rotational DOF related with this direction can be included to transfer the moment to the independent nodes in order to avoid singularity. If necessary, the analysis coordinate frame of the independent nodes has to be changed to align one direction with the hinge axis. On these cases, all the moment will be transferred to the nodes where the rotational DOF is included as independent, as there is no possibility to balance this moment by a set of forces on the independent nodes translational DOF.

As an example, see the following figure. The MX moment cannot be transferred by any force to the independent nodes of the RBE3 (in red). In this case the DOF 4 can be included on the RBE3 independent nodes to transfer the moment.



5. BASIC RULES WHEN CREATING RBE

When a RBE is included on the model there are three basic rules that have to be followed:

- Any rigid body movement of the RBE has to be perfectly defined by the independent DOF. If the DOF are not enough to determine the movement of a rigid body, more DOF have to be added (see the chapter above).
- A dependent DOF cannot be dependent of more than one RBE or MPC.
- A dependent DOF cannot be fixed by an SPC.

