| | | |
|---|---|---|
| www.aersys.com | **AERSYS KNOWLEDGE UNIT** | AERSYS-7016 |

| Author: | Antonino Vicente Rico | Date: | 27/03/2014 |
|---|---|---|---|

| FEM | X | HAND | | LIN | | NOLIN | | BUCK | | FAT | | STATIC | | COMP | X | MET | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|

# DIFFERENCES BETWEEN PCOMP Z0 & CQUAD4 ZOFFS

## 1. INTRODUCTION

The scope of this document is to provide a clarification and a deeper understanding of the two different ways to move the mid plane of the element out of the nodal plane.
Although the use of ZOFFS or Z0 is not recommended, there are some reasons for which it is used for convenience. Some of these reasons are listed below:

- Areas in which two or more components are overlapped. With the purpose of taking advantage of the existing mesh, the desired elements are created in the same position as the initial mesh. This kind of model is really useful for upcoming modifications which will no need the creation of a new mesh.
- Composite parts with thicknesses variation in which is really important to fulfil the tool surface. That is, element faces (taking into account thicknesses, ZOFFS and Z0) must represent the tool face.

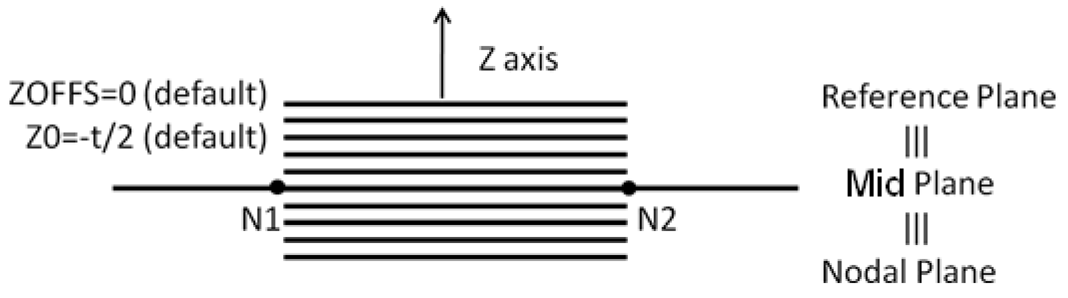
But despite their use, it is really important to take precaution because some of the results are not as they should be expected. All of them will be explained in the next chapters.

## 2. GENERAL CONCEPTS FOR ZOFFS AND Z0

The first thing to do is to explain the different planes or surfaces of a shell element that take relevant importance in the calculation process. All those planes are listed and explained below:

- Nodal Plane: It is the plane or surface (surface only if the element is warped) which is defined by the element nodes.
- Reference Plane: It is the plane or surface in which element forces and laminate matrices are computed.
- Mid Plane: It is the plane or surface which represents the mid plane of the laminate.

For a general element in which there is no ZOFFS and Z0 is the default value, the different planes explained above can be sketched as follows:




Once the previous concepts are explained, let's move onto how Z0 and ZOFFS are modified in the PCOMP and CQUAD4 (as example) cards.

- CQUAD4 ZOFFS

  The ZOFFS value in a CQUAD4 is represented by the first line-ninth field of the corresponding NASTRAN card. It can be seen in the NASTRAN code shown below.

```
CQUAD4  1     1     1     2     4     3     0     22.
```

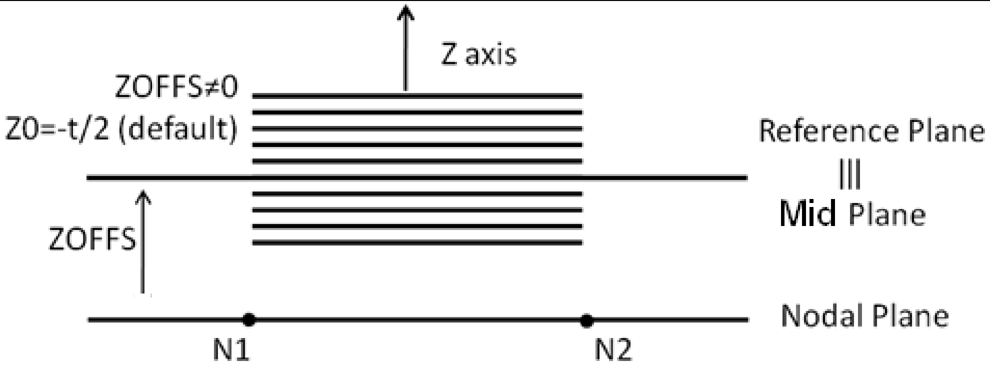
  In this previous example, it is possible to notice that the input offset has a value of 22mm.

  It is important to highlight several aspects of the ZOFFS used:

  1. The ZOFFS is referenced to the Z axis (Z axis of the element); therefore, a positive ZOFFS gives as a result a positive displacement along the Z axis (normal of the element).
  2. The mid plane is moved with the use of a ZOFFS (if Z0 is the default value) with regards to the nodal plane, as a consequence, the element stiffness change. ZOFFS denotes the movement of the reference plane with regards to the nodal plane.
  3. Using ZOFFS is equivalent to locate an RBE2 between the nodes on the nodal plane and the reference plane of the element (coincident with the mid plane if Z0 has its default value), therefore a load eccentricity is created and the stiffness changes.
  4. The laminate matrices ([ABBD]) are calculated on the reference plane and this plane remains on the same position as the mid plane. Therefore, the laminate matrices with ZOFFS and without it are coincident.

  For any ZOFFS value, the configuration of the laminate can be sketched as it is shown in the following image:




As it can be seen, the reference plane has been moved a ZOFFS distance from the nodal plane. Hence, the element has been moved as well so as the mid plane (which is coincident with the reference plane) is not on the nodal plane.

- PCOMP Z0

  When a laminate is used, it is possible to introduce an offset by means of a Z0 into the PCOMP card. The PCOMP NASTRAN card which shows the use of Z0 is shown below.

```
PCOMP   1     21.172
        1     .184   45.    YES   1    .184   -45.   YES
        1     .184   0.     YES   1    .184   0.     YES
        1     .184   45.    YES   1    .184   -45.   YES
        1     .184   90.    YES   1    .184   0.     YES
        1     .184   45.    YES
```

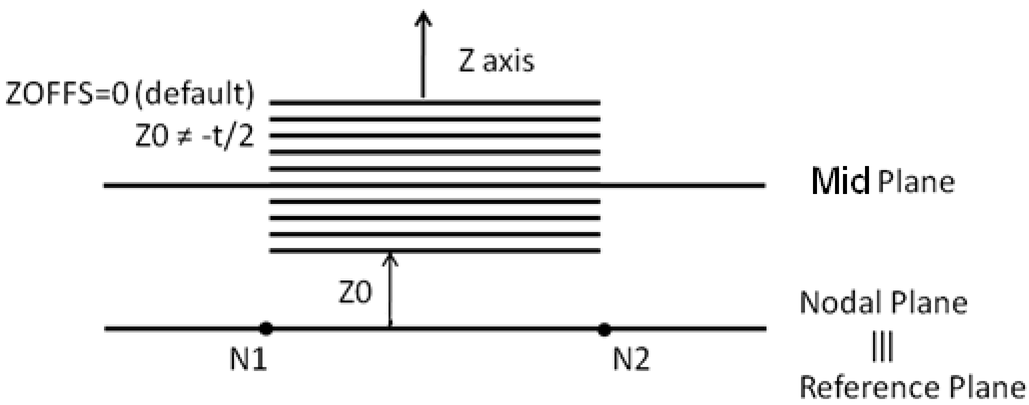
Z0 is introduced in the first line-third field of the PCOMP card. In the previous example, Z0 has a value of 21.172 mm. As a result of the Z0, the mid plane is moved with regards to the nodal plane while the reference plane is not affected. It is important to remark that:

1. The Z0 is referenced to the Z axis (Z axis of the element); therefore, a Z0>-t/2 gives as a result a positive displacement along the Z axis (normal of the element).
2. The mid plane is moved with the use of a Z0 (if Z0 is the default value) with regards to the nodal plane, as a consequence, the element stiffness change. Z0 denotes the distance from the reference plane to the bottom plane of the laminate (normally the tool contacting surface). Referencing the bottom plane or the mid plane has the same effect as they always remain in the same relative distance. If Z0 is changed from the default value, the reference plane moves out of the mid plane.

3. Using Z0 is equivalent to changing the [ABBD] matrices of the laminate. This matrix is calculated on the reference plane. Changing Z0 means offsetting the reference plane with regards to mid plane. Therefore a load eccentricity is created and the stiffness changes, but in a different way as on the ZOFFS case. The load eccentricity on this case is created by means of the bending-membrane coupling matrix [B].
4. On both cases the mid plane can be relocated and the two options are practically equivalent on the stiffness and on the model behaviour. There are differences only on the output for the element forces and stresses on equivalent homogeneous elements (PARAM NOCOMPS 0 or -1).

With the purpose of clarifying the ideas as well as comparing with the previous configurations, the Z0 sketch is shown below:



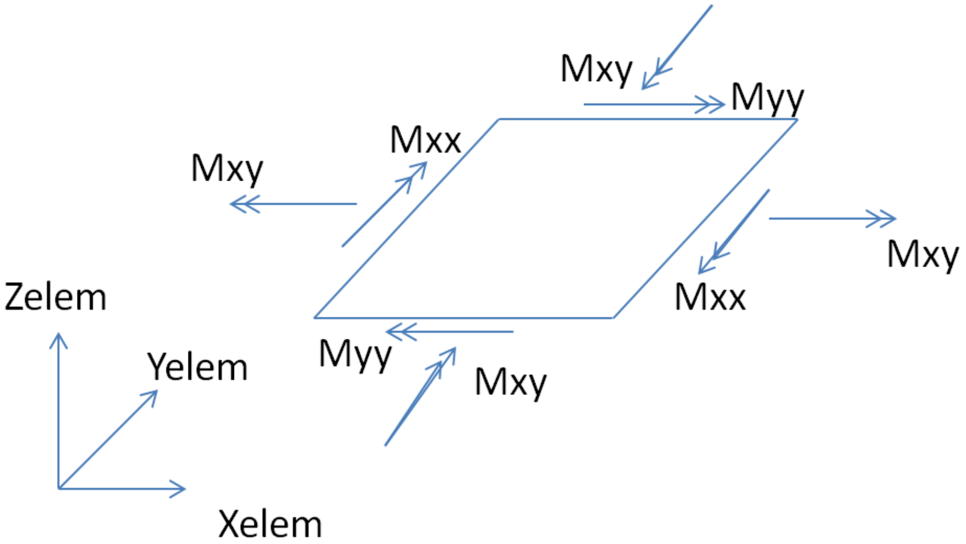

As a consequence of the movement of the mid plane from the reference plane, the laminate matrixes will be modified. This change is going to be explained and clarified in next chapter.

## 3. CLASSICAL THEORY OF LAMINATED PLATES

With the main purpose of understanding the differences between Z0 and ZOFFS, a brief explanation of the classical theory of laminated plates is reported in this chapter.

The first thing to be clarified is the different signs criterion used by NASTRAN and by the laminate theory. As it is well known, NASTRAN sign criterion is in the way presented below:




Moments in Shell elements

While those previous signs are the NASTRAN ones, for the laminate theory it is necessary to consider a change of signs in all the moments. That is due to the forces and moments flows are calculated as equations (1) to (6) show. It is important to highlight that all of these previous equations are referenced to the mid plane; this assumption will be discussed in next chapters.

$$n_{xx} = \int_{-t/2}^{t/2} \sigma_{xx}\, dz \qquad (1)$$

$$n_{yy} = \int_{-t/2}^{t/2} \sigma_{yy}\, dz \qquad (2)$$

$$n_{xy} = \int_{-t/2}^{t/2} \tau_{xy}\, dz \qquad (3)$$

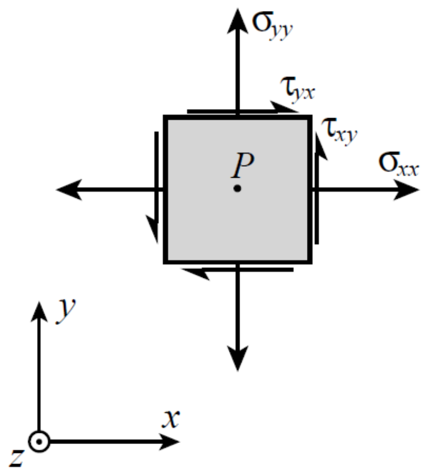
$$m_{xx} = \int_{-t/2}^{t/2} \sigma_{xx}\, z\, dz \qquad (4)$$

$$m_{yy} = \int_{-t/2}^{t/2} \sigma_{yy}\, z\, dz \qquad (5)$$

$$m_{xy} = \int_{-t/2}^{t/2} \tau_{xy}\, z\, dz \qquad (6)$$

Where:
n: Force flows (N/mm)
m: Moment flows (N)
σ,τ: Laminate stresses
t: Laminate thickness

The laminate stresses have the same sign criterion as NASTRAN, which is also shown below.




The previous explanation about the different sign criterion is really important so as to transform the classical laminate matrices to the ones which NASTRAN uses. That is a really critical aspect to take into account in order to calculate the results in the proper way. With that point clarified, it is time to start with the laminate theory.

The flows that have been obtained from equations (1)-(6), can be related with strains and curvatures by mean of equation (7).

$$\begin{Bmatrix} n_{xx} \\ n_{yy} \\ n_{xy} \\ m_{xx} \\ m_{yy} \\ m_{xy} \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \cdot \begin{Bmatrix} \varepsilon_{xx}^0 \\ \varepsilon_{yy}^0 \\ \gamma_{xy}^0 \\ k_{xx} \\ k_{yy} \\ k_{xy} \end{Bmatrix} \tag{7}$$

Where:
ε: Laminate strains
k: Laminate curvatures
[A],[B],[D]:Stiffness submatrix of the laminate

Submatrix [A], [B] y [D] can be calculated using equations (8)-(10)

$$[A] = \int_{-\frac{t}{2}}^{\frac{t}{2}} [Q(z)]\, dz \tag{8}$$

$$[B] = \int_{-\frac{t}{2}}^{\frac{t}{2}} [Q(z)]\, z\, dz \tag{9}$$

$$[D] = \int_{-\frac{t}{2}}^{\frac{t}{2}} [Q(z)]\, z^2\, dz \tag{10}$$

In the previous equations [Q] represents the material matrix, which relates the strains and the stresses as equation (11) shows.

$$\{\sigma\} = [Q]\{\varepsilon\} \tag{11}$$

It is important to notice that the strains on equation (7) are calculated in the mid plane. So, in order to compute the through-thickness strains of the element, it is necessary to use equation (12). This equation relates the strains and curvatures in the mid plane with the strains at any fibre.

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_{xx}^0 \\ \varepsilon_{yy}^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} k_{xx} \\ k_{yy} \\ k_{xy} \end{Bmatrix} \tag{12}$$

For static analysis, the material matrix can be assumed as constant, therefore, it can be placed out of the integral, giving as a result a summation (equations (13)-(15)).

$$[A] = \sum_{k=1}^{N} [Q]_k\, (z_{k+1} - z_k) \tag{13}$$

$$[B] = \frac{1}{2}\sum_{k=1}^{N} [Q]_k\, (z_{k+1}^2 - z_k^2) \tag{14}$$

$$[D] = \frac{1}{3}\sum_{k=1}^{N} [Q]_k\, (z_{k+1}^3 - z_k^3) \tag{15}$$

Where:
$z_k$ represents the distance between the bottom of the z$^{th}$ ply to the plane in which laminate matrixes are computed. The plane where the matrices are computed is the mid plane for the laminate theory and it is the reference plane for NASTRAN.
N is the number of plies within the laminate.

It is possible to relate the laminate matrixes to the ones used by NASTRAN. The matrices used by NASTRAN are the material matrixes from the PSHELL card (MID1, MID2, MID3, MID4). The desired relation is given by equations (16)-(18).

$$[MID1] = \frac{1}{t}[A] \tag{16}$$

$$[MID2] = \frac{12}{t^3}[D] \tag{17}$$

$$[MID4] = -\frac{1}{t^2}[B] \tag{18}$$

For an insight of the previous relations, please, read Aersys Knowledge Unit 7001.

It is important to notice that in equation (18) there is a negative sign. This is because of the different sign criterion between NASTRAN and the laminate theory, which has been explained previously.

## 4. OVERVIEW OF THE CALCULATION PROCESS

Following, there is a brief and general explanation of how NASTRAN computes the different results.

1. Starting from the finite elements theory, the structure analysis is set in terms of forces and displacements which are related by the stiffness matrix.

2. When the system of equations is solved, the displacements for the DOF of the structure are obtained. With these displacements and the stiffness matrix of the element it is possible to obtain the element forces on each element. For shell elements, element forces are given per unit of length; therefore they represent the flows of the element.

3. Once the flows are obtained, the strains and the curvatures can be computed in the mid plane of the element at which they have been referenced. They are related by means of equation (7)

4. Once the strains and curvatures have been obtained in the mid plane, it is possible to compute the strains at each ply using equation (12).

5. The resulting strains from the previous equation are referenced to the element coordinate system. Therefore, it is necessary to transform each of them to its appropriate coordinate system. After changing the strains of coordinate system, it is possible to compute the stresses in each ply by means of the material matrix which is given by equation (11)

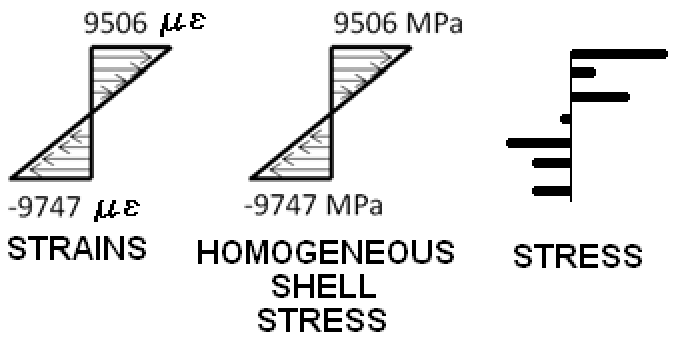
 

It is important to notice that these previous stresses (step 5) have (in general) different values from the ones obtained as stresses in homogeneous shell elements, which are computed on the top and bottom of the laminate. The difference is not only consequence of the different direction between the ply and element system but also from the way in which they are calculated.
To compute the stresses in homogeneous shell elements, NASTRAN has as input the element forces and uses equations (19)-(21).

$$\sigma_{xx} = \frac{n_{xx}}{t} + m_{xx}\frac{12}{t^3}z \tag{19}$$
$$\sigma_{yy} = \frac{n_{yy}}{t} + m_{yy}\frac{12}{t^3}z \tag{20}$$
$$\tau_{xy} = \frac{n_{xy}}{t} + m_{xy}\frac{12}{t^3}z \tag{21}$$

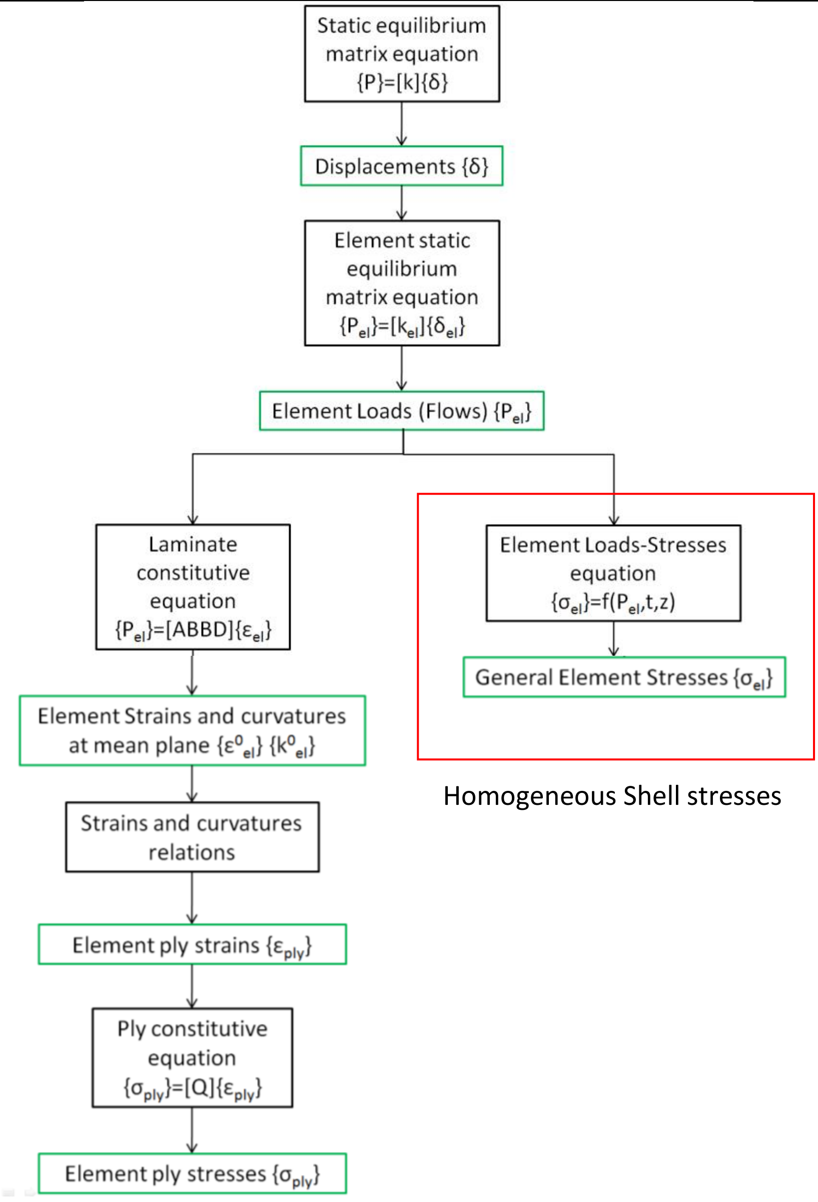
In the previous equations, z represents the distance between the plane in which flows are computed and the desires plane in which stresses are obtained. The equations above are simply the equilibrium of moments and forces with a linear distribution of stresses through the thickness.

As it is easy to see, the general stresses of the element have been computed as they were continuous. This kind of behaviour is only valid for homogeneous materials in which stresses are continuous. As a consequence of it, for most of laminates this hypothesis is not true and results are “incorrect”. The strains are always continuous (basic hypothesis) but the stresses are not (they are in general discrete values as the strains are continuous and the stiffness (E) of each ply is different depending on the orientation).




Below, a scheme of the calculation process is displayed:




## 5. SUMMARY

- Both options (ZOFFS on elements and Z0 on properties) are practically equivalent in terms of global behaviour of the model and stiffness. With the same loading of the model, the same displacements will be obtained (differences<1%). The outputs for all the magnitudes (stresses, strains, stc) are practically identical except the element forces and homogeneous shell element stresses.

- When only Z0 is used, element forces are not calculated in the mid plane unless Z0=-t/2, because the element forces are computed on the reference plane and the Z0 moves the mid plane out of the reference plane. Therefore, they are not the expected values as they are not in concordance with the classical theory of structures because they are not applied in the mid plane. When only ZOFFS is used, the reference plane and the mid plane are coincident and the value of the element forces is the expected one.

- Using Z0 different to the default value changes the [B] and [D] matrixes of the laminate. Changing the [D] matrix, the effect of the displacement of the mid plane is accounted on the inertial properties of the laminate. Changing the [B] matrix the effect of the load eccentricity is accounted on the bending-membrane coupling. This can also be obtained by using ZOFFS, but in this case the laminate matrixes remain the same that without ZOFFS and the change on the inertia and the load eccentricity is got by a kind of internal RBE2.

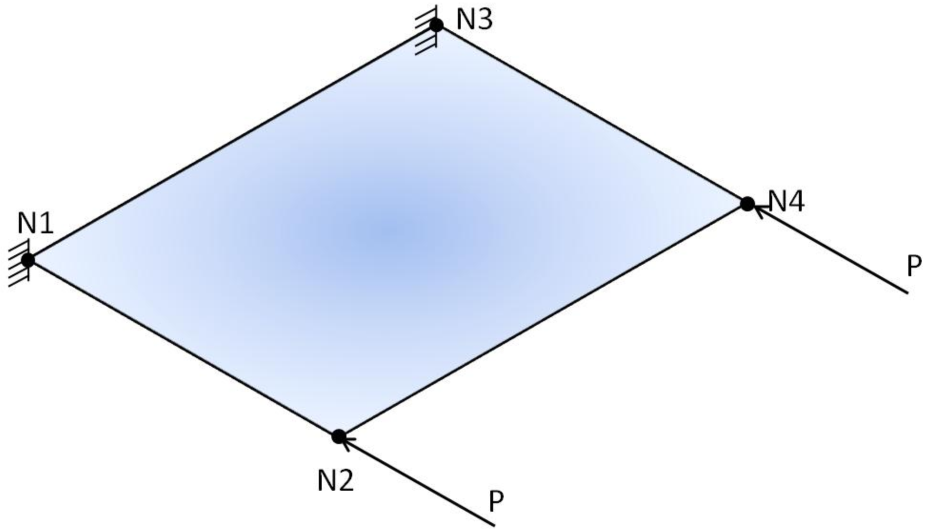
- The stresses in homogeneous shell elements are computed based on the element forces. As the element forces are calculated on the reference plane, and the reference plane is different to the mid plane if Z0 is not equal to –t/2 (default), the homogeneous shell stresses do not take into account the offset implemented on the Z0. They are calculated as if the shell element were not offset by means of Z0. That is, with the flows of the nodal plane as if they were applied on the mid plane. Anyway, the homogeneous element stresses are not relevant values if the material is not homogeneous (because otherwise the hypothesis of continuous stresses is not correct and they are simply a result of equilibrium assuming a linear distribution of stresses through the thickness).

## 6. DETAILED EXAMPLE PCOMP Z0 & CQUAD4 ZOFFS

This chapter will explain the difference between the two configurations which are discussed in the present document. This analysis is going to be carried out by means of a random laminate in which both, ZOFFS and Z0 are tested. The laminate chosen for this purpose has the following laminate sequence (+45/-45/0/0/+45/-45/90/0/+45).

In order to simplify the analysis and obtain clear results, only one element is going to be analysed. It will be a CQUAD4 which is clamped in one of its sides and has a compression load in the opposite side. It is represented in the following image.



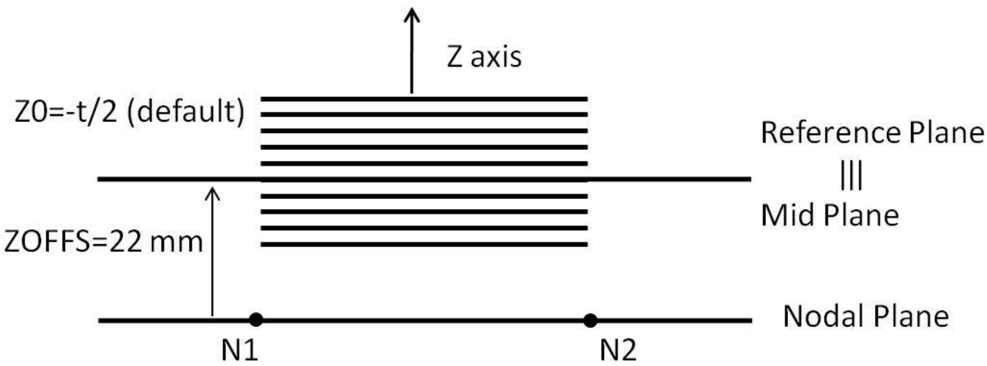

- **<u>CQUAD4 ZOFFS</u>**

The first study to be done is the ZOFFS one.
For this case, the bulk data which has been used is displayed below:

```
SOL 101
CEND

SUBCASE 1
…
BEGIN BULK
…
PCOMP    1
       1     .184    45.     YES    1     .184    -45.    YES
       1     .184    0.      YES    1     .184    0.      YES
       1     .184    45.     YES    1     .184    -45.    YES
       1     .184    90.     YES    1     .184    0.      YES
       1     .184    45.     YES

CQUAD4   1      1      1      2      4      3      0      22.

MAT8     1     154000. 8500.  .35    4200.  4200.  2500.

GRID     1             0.     0.     0.
```

```
GRID    2           1000.   0.     0.
GRID    3            0.    1000.   0.
GRID    4           1000.  1000.   0.

SPC1    1     123456  1     3

FORCE   1      2      0     100000. -1.     0.     0.
FORCE   1      4      0     100000. -1.     0.     0.
```

As it can be appreciated, a ZOFFS of 22 mm has been introduced, hence the configuration of the element will be as follow:




The matrix [Q] for an orthotropic material is shown in equation (22).

$$[Q] = \begin{bmatrix} \frac{E_1}{1-\nu_{12}\nu_{21}} & \frac{E_1\nu_{21}}{1-\nu_{12}\nu_{21}} & 0 \\ \frac{E_2\nu_{12}}{1-\nu_{12}\nu_{21}} & \frac{E_2}{1-\nu_{12}\nu_{21}} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \quad (22)$$

Where it is important to remember the following relation:

$$E_2\nu_{12} = E_1\nu_{21} \quad (23)$$

For this example, the different material's properties have the values shown below.

$E_1$=154000 MPa
$E_2$=8500 MPa
$\nu_{12}$=0.35
$\nu_{21}$=0.019318

$G_{12}$=4200 MPa

To calculate the laminate matrixes [A], [B] and [D], the $z_k$ takes values from –t/2 for k=1 to t/2 for k=N+1. It is important to remember that the reasons of these values is that the reference plane is coincident with the mid plane because Z0=-t/2 (default value).

For the case under study, matrix [Q] has the following values:

$$[Q_0] = \begin{bmatrix} 155048.3 & 2995.25 & 0 \\ 2995.25 & 8557.86 & 0 \\ 0 & 0 & 4200 \end{bmatrix}$$

These previous values are only valid for the 0° plies. For other directions, it is necessary to transform the matrix. The matrix used in these example are displayed below:

$$[Q_{45}] = \begin{bmatrix} 46599.18 & 38199.18 & -36622.62 \\ 38199.18 & 46599.18 & -36622.62 \\ -36622.62 & -36622.62 & 39403.92 \end{bmatrix}$$

$$[Q_{45}] = \begin{bmatrix} 46599.18 & 38199.18 & 36622.62 \\ 38199.18 & 46599.18 & 36622.62 \\ 36622.62 & 36622.62 & 39403.92 \end{bmatrix}$$

$$[Q_{90}] = \begin{bmatrix} 8557.86 & 2995.25 & 0 \\ 2995.25 & 155048.3 & 0 \\ 0 & 0 & 4200 \end{bmatrix}$$

Using equations (13)-(15), it is possible to compute the laminate matrixes.

$$[A] = \begin{bmatrix} 130032.56 & 37347.74 & 6738.56 \\ 37347.74 & 76124.07 & 6738.56 \\ 6738.56 & 6738.56 & 39342.8 \end{bmatrix}$$

$$[B] = \begin{bmatrix} -2575.85 & -2383.73 & 2479.79 \\ -2383.73 & 7343.31 & 2479.79 \\ 2479.79 & 2479.78 & -2383.73 \end{bmatrix}$$

$$[D] = \begin{bmatrix} 26294.52 & 10435.63 & 5038.1 \\ 10435.63 & 17016.79 & 5038.1 \\ 5038.1 & 5038.1 & 10891.56 \end{bmatrix}$$

After running the NASTRAN code, element forces are extracted from the .f06 file.

F O R C E S   I N   Q U A D R I L A T E R A L   E L E M E N T S   ( Q U A D 4 )      OPTION = BILIN

MEMBRANE FORCES:
FX: -2.000000E+02
FY: -1.427648E+02
FXY: -1.122072E-02

BENDING MOMENTS:
MX: -4.400000E+03
MY: -3.847551E+02
MXY: -8.705566E+01

TRANSVERSE SHEAR FORCES:
QX: 5.329071E-14
QY: -1.213519E-02

With the flows extracted and the equation (7), it is easy to compute the strains and the curvatures.
It is really important to remember that the flows from the laminate theory and from NASTRAN have different sign criterion. Therefore, in order to obtain the results as the ones which are in the .f06 (NASTRAN), it is necessary to change the sign of the matrix [B]. As [D] sign is not changed, the curvatures are obtained with the NASTRAN criterion.

Once matrix [B] has been changed of sign, it is only necessary to invert the ABBD matrix to obtain the desired results. Performing that calculation, the strains and curvatures are computed giving as a result the following values:

$$\begin{Bmatrix} \varepsilon_{xx}^0 \\ \varepsilon_{yy}^0 \\ \gamma_{xy}^0 \\ k_{xx} \\ k_{yy} \\ k_{xy} \end{Bmatrix} = \begin{Bmatrix} -2.9660\mathrm{E}-03 \\ 1.9523\mathrm{E}-02 \\ -1.3425\mathrm{E}-02 \\ -2.2172\mathrm{E}-01 \\ 1.0458\mathrm{E}-01 \\ 5.2900\mathrm{E}-02 \end{Bmatrix}$$

These values are exactly the same as the ones from the .f06

With the above values it is possible to calculate the strains through the thickness. And also the strains and stresses at each ply. This process has been explained in the previous chapters. It is important to notice and remember that the curvatures have been calculated according to the NASTRAN sign criterion. Therefore it is necessary to change their sign so as to use equation (12).

The last results to be computed are the stresses in the element. As it was explained in the previous chapter, these stresses are computed in the element forces. Using equations (19) to (21), it is possible to compute the stresses in the bottom of the element which is set at -0.828 mm below the reference plane.
Computing that, the following stresses are obtained:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix} = \begin{Bmatrix} -9747.60 \\ -928.02 \\ -190.48 \end{Bmatrix}$$

As it has been considered in this documents, the moments must be changed of sign so as to obtain the properly values of strains.
Please, remember that these values are worthless since the element is not homogeneous.

- **PCOMP Z0**

Now, let’s move onto the PCOMP Z0 example.

The NASTRAN code used in this example is the same as the one used in the previous example as far as ZOFFS are removed and the Z0 value changed. The value of Z0, has to be the proper one to translate the mid plane into the same position as the previous example. That is Z0= ZOFFS –t/2

```
SOL 101
CEND

SUBCASE 1
…
BEGIN BULK
…
PCOMP   1     21.172
        1     .184   45.    YES   1    .184   -45.   YES
        1     .184   0.     YES   1    .184   0.     YES
        1     .184   45.    YES   1    .184   -45.   YES
        1     .184   90.    YES   1    .184   0.     YES
        1     .184   45.    YES

CQUAD4  1     1      1      2     4    3      0

MAT8    1     154000. 8500. .35   4200. 4200. 2500.

GRID    1            0.     0.    0.
GRID    2            1000.  0.    0.
GRID    3            0.     1000. 0.
```

```
GRID    4          1000.   1000.   0.

SPC1    1     123456  1      3

FORCE   1      2      0      100000. -1.     0.      0.
FORCE   1      4      0      100000. -1.     0.      0.
```

The material matrix [Q], is the same for both example, but matrixes [A], [B] and [D] change as a consequence of Z0. As Z0 changes, $z_k$ also do the same, and as consequence of the quadratic and cubic influence in [B] and [D], they change.

With all the previous considerations, it is possible to calculate [A], [B] and [D]

$$[A] = \begin{bmatrix} 130032.56 & 37347.74 & 6738.56 \\ 37347.74 & 76124.07 & 6738.56 \\ 6738.56 & 6738.56 & 39342.8 \end{bmatrix}$$

$$[B] = \begin{bmatrix} 2858140.72 & 819266.72 & 150728.15 \\ 819266.72 & 1682073. & 150728.15 \\ 150728.15 & 150728.15 & 863158.1 \end{bmatrix}$$

$$[D] = \begin{bmatrix} 62848721.62 & 17981861.44 & 3375612.85 \\ 17981861.44 & 37184175.52 & 3375612.85 \\ 3375612.85 & 3375612.85 & 18947927.72 \end{bmatrix}$$

The process to obtain the results is the same as the one used for the first example. So now the element forces are obtained from the .f06.

F O R C E S   I N   Q U A D R I L A T E R A L   E L E M E N T S   ( Q U A D 4 )      OPTION = BILIN

MEMBRANE FORCES:
FX: -2.000000E+02
FY: -1.427512E+02
FXY: -1.116929E-02

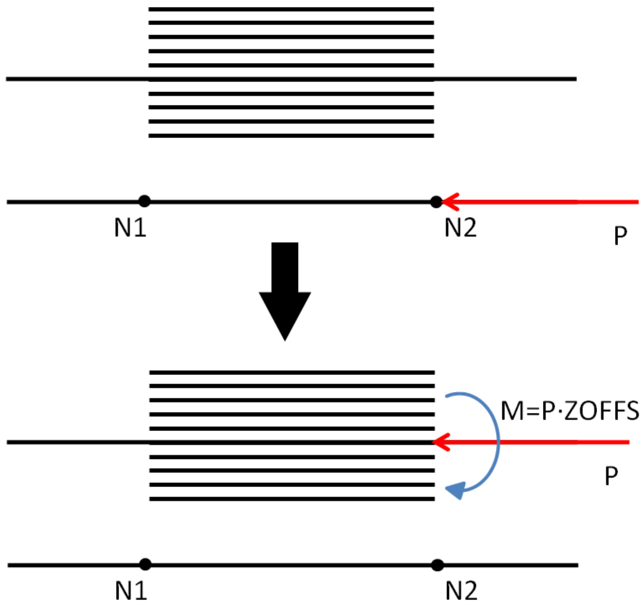
BENDING MOMENTS:
MX: -4.656613E-09
MY: 2.755814E+03
MXY: -8.707347E+01

TRANSVERSE SHEAR FORCES:
QX: -7.275958E-12
QY: -4.679089E-02

The first thing to notice is the difference in the fluxes for both examples. These differences are mainly focused on the bending moments, and their reason is going to be explained below.

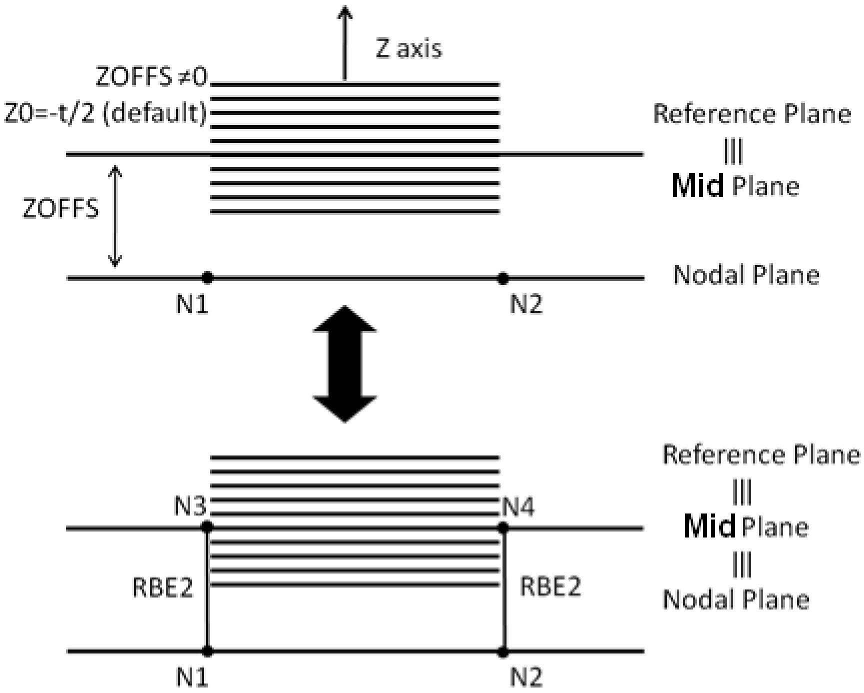
As it was mentioned in the second chapter, element forces are computed in the reference plane. For the ZOFFS example the reference plane is coincident with the mid plane, both have been translated from the nodal plane. As loads are applied in the nodal plane, a moment is generated when they are moved to the reference plane. That is the reason why the element forces have different values for both examples.

A schematic drawing of the equivalent moments is displayed below.




An easy and really straightforward way to understand the previous behaviour of the ZOFFS is to equate it with a non-offset element translated by means of RBE2. This can be seen in the following image.



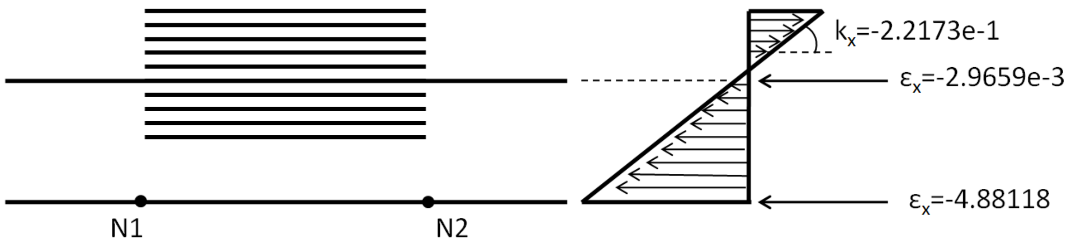

Coming back to the calculation process, once flows are obtained it is possible to carry on the calculation process and obtain the strains and curvatures by means of equation (7). The values of this calculation are displayed below:

$$\begin{Bmatrix} \varepsilon_{xx}^0 \\ \varepsilon_{yy}^0 \\ \gamma_{xy}^0 \\ k_{xx} \\ k_{yy} \\ k_{xy} \end{Bmatrix} = \begin{Bmatrix} -4.8812\mathrm{E}+00 \\ 2.3204\mathrm{E}+00 \\ 1.1496\mathrm{E}+00 \\ -2.2174\mathrm{E}-01 \\ 1.0458\mathrm{E}-01 \\ 5.2863\mathrm{E}-02 \end{Bmatrix}$$

As well as the strains and curvatures of the first example, these ones are exactly the same as the ones reported in the .f06 file.

Comparing the values for both examples it is possible to note that while the strains are different, the values of the curvatures are the same.

As it has said before, the strains and deformations are obtained in the reference plain, that is the reason why the curvatures are the same while the deformations are different. To clarify the different strains, a schematic view of the strains distribution has been given below. As it is easy to appreciate the only different is the reference plane in which strains have been computed.




After computing the deformations, it is possible to obtain the strains and stresses for each ply. Their values are exactly the same for both examples. That is because the laminate matrixes [A], [B] and [D] are referenced to the reference plane. Therefore in these three matrixes the distance from the reference plane to the mid plane has been taken into account, which is the equivalence to the moments introduced by the ZOFFS in the other example.

Regarding to the stresses in quadrilateral elements, it is possible to detect two different errors.

The first error is the same as the one in the previous example. That is, the assumption of a continuous stresses is only valid for homogeneous elements. As it is well known, this condition is not fulfilled for most of the laminates.

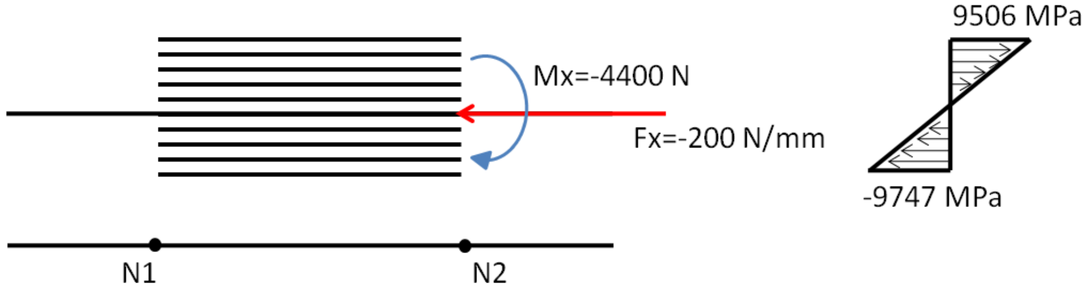
But there is another error; this is a consequence of the position of the reference plane in which element forces are referenced. As it has been explained in this document, element forces are referenced to the reference plane and in this example when Z0 is used, the element forces do not take into account the moment due to the offset.

It is possible to fix this error although it has to be made manually. To fix it, it is only necessary to move the element forces from the reference plane to the mid plane taking into account the moments generated. After that, the stresses are computed using equations (19)-(21). Note that this setting only fixes the last one so the first error of homogeneous plate will remain.

The two examples are displayed bellow in a schematically way so as to notice the differences.

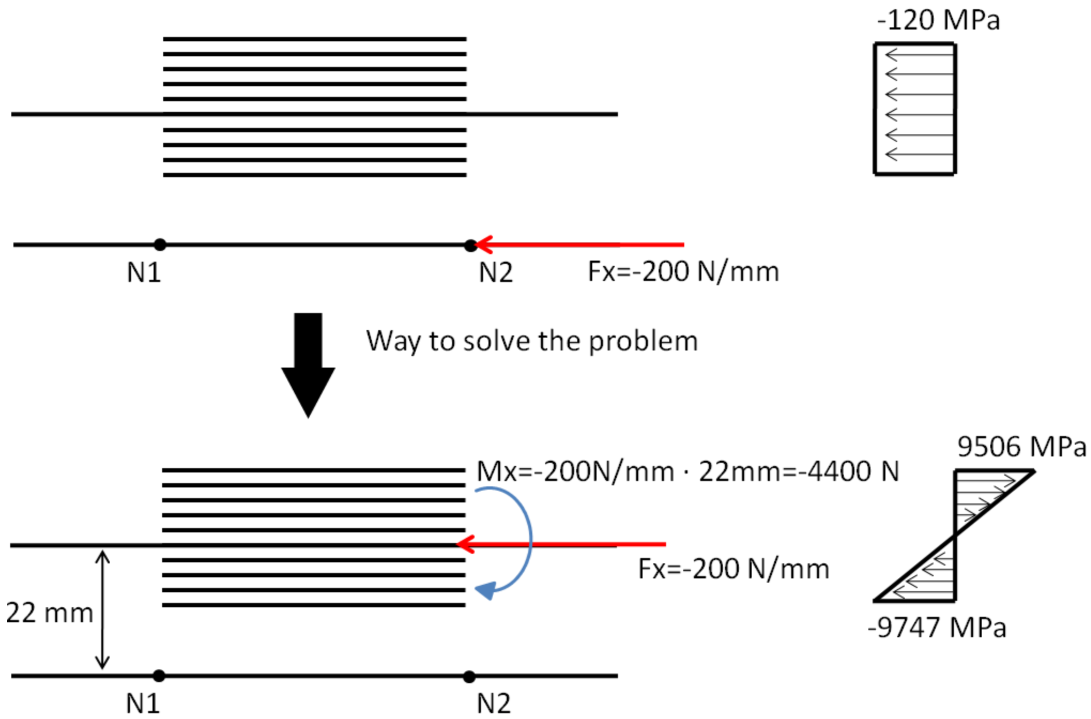
 

**ZOFFS:**




In the previous image the element forces and stresses for the x direction are displayed. This is for the ZOFFS example in which element forces are referenced to the reference plane.

**Z0:**




In this image, the Z0 example is plotted. The image on the top represents the NASTRAN values, whiles the bottom one is the configuration fixed.

As it is possible to see, results from the first configuration are completely different from the ZOFFS example values. But for the second configuration in which forces are moved, the values are the same as the ZOFFS example.