CONSTRAINTS FOR STATIC AND BUCKLING ANALYSIS

1. INTRODUCTION

Linear buckling analysis on NASTRAN is performed with boundary conditions which need not be the same as in the static analysis. The difference between both boundary conditions is going to be explained in this document.

The main purpose for using different boundary conditions is to perform a conservative analysis regarding an analysis performed with the static boundary conditions. The reason of performing a conservative analysis is that sometimes boundary conditions cannot be fully proved. As an example, simple supported boundary condition should be applied instead of clamping condition.

Before continuing with the explanation, it is important to highlight that the buckling analysis is performed to study the buckling of the static load case. So it is the static load case the one which define the buckling case, applying on it different kind of boundary conditions to do it more or less conservative. This aspect is deeply explained in the following chapters.

2. THEORETICAL BACKGROUND

Buckling is a consequence of stiffness changes due to the existing displacements. To take into account these stiffness changes, higher order terms of the force-displacement relationships must be retained. In linear buckling only the first order contribution is considered.

Therefore in linear buckling the first order differential stiffness matrix must be included into the static load-displacements equation. Although considering this effect, NASTRAN does not implement other effects like follower forces or small deflections in linear analysis.

To understand the origin of the differential stiffness matrix a simple process is going to be explained.

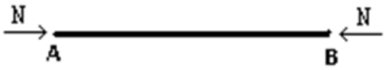

Figure 1

The equation which represent the behavior of a static structure is given by equation (1) in which $\{p\}$ is the load vector, $[K]$ is the stiffness matrix and $\{u\}$ is the displacement vector.

$$\{p\} = [K]\{u\} \qquad (1)$$

When the structure is loaded, displacements appear and therefore a geometrical change happens. As a consequence of this, the loads change their orientation regarding the non-deformed status as is shown in Figure 2.

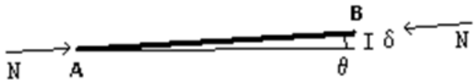

Figure 2

Taking into account the influence of the re-positioned nodes, the stiffness matrix can be re-written as a function of the loads (linear or non-linear in general) obtained and hence equation (1) should be rewritten as:

$$\{p\} = [K(\{p\})]\{u\}$$

The stiffness matrix in general is a very complex function of the load vector {p}, but using the classical local developments of the function (Taylor) the following approach can be written:

$$[K(\{p\})] = [K(\{0\})] + [\Delta K'(\{p\})]\mu + [\Delta K''(\{p\})]\mu^2 + \cdots$$

Where the factor μ is a factor multiplying the loads (for this case is 1.0 but can be used to locally calculate the value of the stiffness matrix with escalated loads, or in a different way unitary values can be used to get the matrices [ΔK] and μ can be used to match the loads).

Neglecting the second order terms of the equation above, the resulting expression represents the first order behavior of the structure. The equation (1) can be written as:

$$\{p\} = [K(\{0\})]\{u\} + [\Delta K'(\{p\})]\mu\{u\} \qquad (2)$$

It is important to notice that the differential stiffness matrix ($[\Delta K'(\{p\})]$) is a function of the load. The reason of this is that loads produce displacements in the structure which determine the differential stiffness matrix. As a consequence of this dependence a non-linear equations system is obtained.

$$\{p\} = [[K(\{0\})] + [\Delta K'(\{p\})]\mu]\{u\}$$

When the determinant of the matrix is null, the equation system is compatible but not determinate. In the studied problem this condition is achieved for some load values which are the critical buckling loads ($\{p\}\cdot\mu$). For this values the determinant of the first linear approach of the stiffness matrix is null:

$$|[[K(\{0\})] + [\Delta K'(\{p\})]\mu]| = 0$$

This problem is an eigenvalue problem with a set of eigenvector $\{\varphi\}$ associated, such that they fulfill the following equation:

$$\{0\} = [[K(\{0\})] + [\Delta K'(\{p\})]\mu]\{\varphi\}$$

Therefore, to determine the stability of a structure the eigenvalues and eigenvectors of the matrix shown in the equation above have to be obtained. After the eigenvalues analysis is performed, critical loads and buckling modes can be obtained. Note that the eigenvalue analysis is performed after imposing the buckling boundary conditions which constrain the buckling modes (but not the static solution).

To obtain critical loads from the eigenvalues obtained in the equations it is only necessary to apply equation (5).

$$\{p_{cr_i}\} = \mu \cdot \{p\} \qquad (5)$$

The way in which linear buckling analysis is implemented in NASTRAN is going to be explained in the following lines:

1.- NASTRAN writes the unconstrained stiffness matrix and writes the following equation:

$$\{p\} = [K(\{0\})]\{u\}$$

2.- NASTRAN applies the SPC of the static subcase obtaining a new system of equations with some rows and columns removed (the ones corresponding with DOF with a SPC applied on the static subcase). The SPC of the static subcase is therefore used on the static subcase to find out the displacement field of the static solution

$$\{\bar{p}\} = [\bar{K}(\{0\})]\{\bar{u}\}$$

Where the hat on the symbols means that some rows and columns have been removed on the vectors and matrices due to the static subcase constraints.

3.- NASTRAN solves the system and gets the displacements for the static subcase:

$$\{\bar{p}\} = [\bar{K}(\{0\})]\{\bar{u}\} \quad \rightarrow \{\bar{u}\} \rightarrow \quad \{u\} \tag{6}$$

4.- NASTRAN calculates the unconstrained stiffness matrix and the unconstrained differential stiffness matrix with the displacement field obtained on the previous step

$$\begin{gathered}[K(\{0\})] \\ \{u\} \rightarrow \{p\} \rightarrow [\Delta K'(\{p\})]\end{gathered}$$

5.- NASTRAN writes the unconstrained problem of eigenvalues/eigenvectors:

$$\{0\} = [[K(\{0\})] + [\Delta K'(\{p\})]\mu]\{\varphi\}$$

6.- NASTRAN applies the boundary conditions of the buckling subcase on this problem, removing some rows and columns from the matrices and vectors

$$\{\bar{\bar{0}}\} = [[\bar{\bar{K}}(\{0\})] + \left[\overline{\overline{\Delta K'}}(\{p\})\right]\mu]\{\bar{\bar{\varphi}}\}$$

Where the double hat on the symbols means that some rows and columns have been removed on the vectors and matrices due to the buckling subcase constraints.

7.- NASTRAN solves this problem obtaining the eigenvalues and eigenvectors

It is recalled that the buckling boundary conditions do not need to be the same as the static ones. The static constrains are used to get the static solution while the buckling constraints are used to constrain the buckling shape, not the static deformation.

3. FORCES OR DISPLACEMENTS LOADS

As it has seen in the previous chapter, the differential stiffness matrix is obtained from the static displacements. Therefore to perform the buckling analysis, a displacement field is necessary which can be obtained by different ways. In the usually way the structure is loaded with forces, and the displacements are obtained after the analysis. But it is also quite usual to load a structure with enforced displacements. One of the most common cases of this use is a detailed analysis of a part of the structure.

To clarify both possibilities their NASTRAN code is presented below.

```
SOL 105
…
CEND
…
SPC = 1
…
SUBCASE 1
LOAD = 1
…
SUBCASE 2
METHOD = 1
…
FORCE   1     5     0     100.   0.    -.707107-.707107
…
```

```
SOL 105
…
CEND
…
SPC = 1
…
SUBCASE 1
LOAD = 2
…
SUBCASE 2
```

```
METHOD = 1
…
SPCD    2    5    1    100.   5    2    100.
SPC1    1    123    5
…
```

4. BOUNDARY CONDITIONS

In this chapter, the differences between boundary condition in static and buckling cases are going to be explained.

As it has seen in the theoretical chapter, NASTRAN solves the static case to obtain the displacements and hence the differential stiffness matrix. So the boundary conditions of the static case constrain the components of the differential stiffness matrix. After obtaining the previous matrix, buckling analysis can be performed. The Boundary conditions in buckling analysis do not need to be the same as the static case.

It is important to remember that the aim of a buckling analysis is to analyze the buckling behavior of a structure constrained with certain boundary condition and loaded with the loads of the static case.

If it is possible to assure that the static boundary conditions are the same as the real boundary condition, then static and buckling boundary conditions can be the same. The NASTRAN code for this case is shown below.

```
SOL 105
…
CEND
…
SPC = 1
…
SUBCASE 1
LOAD = 2
…
SUBCASE 2
METHOD = 1
…
```

But sometimes it is impossible to assure the relation between real and static boundary conditions. So, in these cases different boundary conditions can be applied for static and buckling cases. An example of NASTRAN code for this kind of analysis is shown below

```
SOL 105
…
CEND
…
SUBCASE 1
LOAD = 2
SPC = 1
…
SUBCASE 2
METHOD = 1
SPC = 2
…
```

It is important to highlight that the purpose of different boundary conditions for static and buckling analysis is to perform a more conservative analysis. So the boundary conditions of the buckling case must not change too much regarding the static ones. If this condition was not met, the buckling case would analyze a different problem respect to the static case.

A remark should be done about the difference between boundary conditions on static and buckling case.

When the differential stiffness matrix is calculated with the static boundary conditions, it is possible to lose some contributions regarding a differential stiffness matrix calculated with the buckling boundary conditions. This stiffness lost is related with the DOF which are constrained in the static case but are free in the buckling case. In general the loss of these contributions provides a more conservative analysis.

To clarify this issue the structure shown in Figure 3 is going to be analyzed.

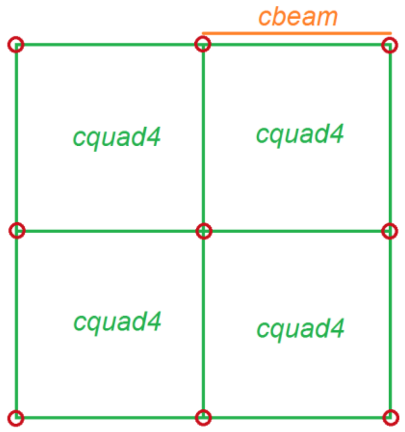

Figure 3

As can be seen in Figure 3, the structure is constituted by four cquad4 and one cbeam element.

In the static case clamped boundary conditions are imposed in the four edges of the plate. But in the buckling case simply supported conditions are imposed instead of cantilever conditions. Both cases are shown in Figure 4.

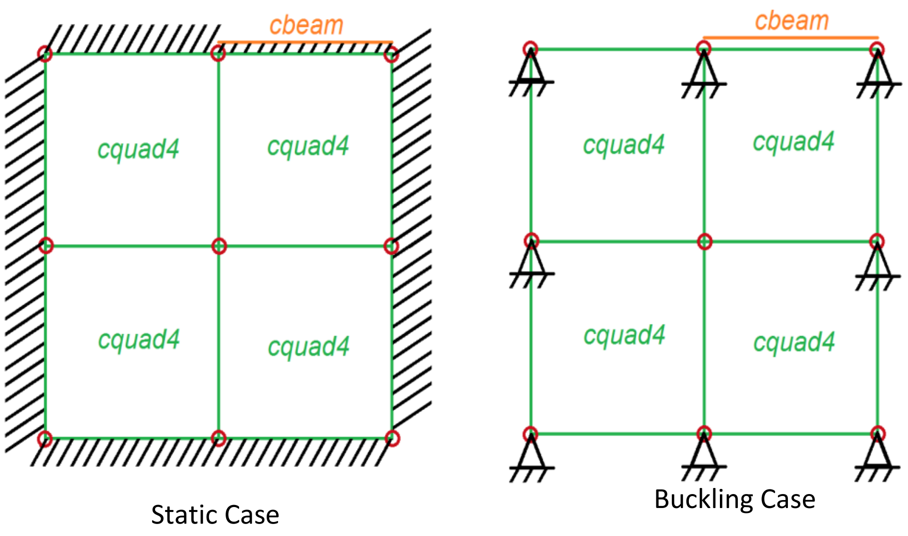

Figure 4

When the static case is analyzed only the six displacements corresponding to the 6 DOFs of the central node are nonzero. As a consequence of this static displacements, differential stiffness contributions are obtained for those elements which are joined with the central node. But for the cbeam element no differential stiffness contribution is obtained. So the differential stiffness matrix only takes into account the contributions corresponding to the cquad4 elements (elements attached to the central node).

When the buckling analysis is performed, the 6 DOFs of the central node and the rotational DOFs of the boundary nodes are both considered. Therefore the linear static matrix takes into account the rotational stiffness contributions of the cbeam while the differential stiffness matrix does not consider the contributions of the cbeam.