

SHELL MESH INTERPOLATION WITH RSPLINES

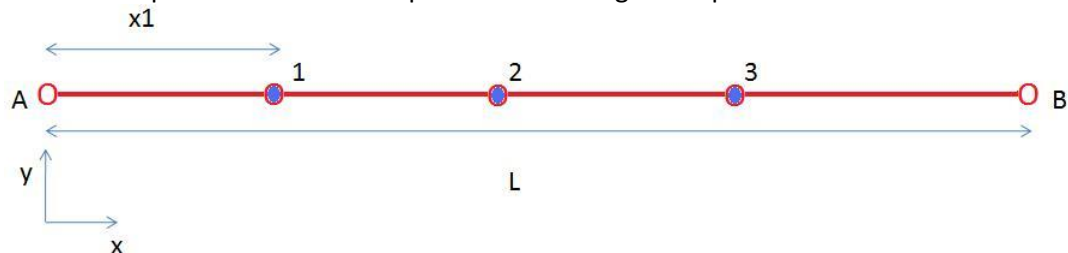
1. INTRODUCTION

The RSPLINES on NASTRAN are a special case of MPC commonly used to interpolate meshes with a different mesh density on each side. They are widely used due to the simplicity of the definition. However, several problems arise when using RSPLINES to interpolate meshes.

This document explains all the problems found so far when using RSPLINES and the way to fix these problems in order to correctly interpolate the displacements field.

2. THEORETICAL BEHAVIOUR OF RSPLINES

The RSPLINE works as a beam element between each pair of independent nodes A and B. The main equations of a beam impose the following assumptions:



- In the direction of the RSPLINE (x) the displacements and rotations are linearly interpolated:

$$u_x^{node\ 1} = \frac{x_1}{L}(u_B - u_A) + u_A$$

$$\theta_x^{node\ 1} = \frac{x_1}{L}(\theta_B - \theta_A) + \theta_A$$

This means that the axial and torque behavior of the RSPLINE is as a beam or rod.

- In the directions transverse to the RSPLINE the displacements are interpolated as a cubic curve.

$$u_y^{node\ 1} = a_y(x_1)^3 + b_y(x_1)^2 + c_y x_1 + d_y$$

$$u_z^{node\ 1} = a_z(x_1)^3 + b_z(x_1)^2 + c_z x_1 + d_z$$

- In the directions transverse to RSPLINE the rotations are interpolated as a quadratic curve. This quadratic curve is the derivate of the displacements (as in a beam).

$$\theta_z^{node\ 1} = 3a_y(x_1)^2 + 2b_y x_1 + c_y$$

$$\theta_y^{node\ 1} = 3a_z(x_1)^2 + 2b_z x_1 + c_z$$

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- This last two assumptions lead to a system of four equations and four constants on each transverse direction when the equations are applied to the nodes A and B:

On y direction:

$$u_y^{node A} = d_y$$

$$u_y^{node B} = a_y(L)^3 + b_y(L)^2 + c_yL + d_y$$

$$\theta_z^{node A} = c_y$$

$$\theta_z^{node B} = 3a_y(L)^2 + 2b_yL + c_y$$

On z direction:

$$u_z^{node A} = d_z$$

$$u_z^{node B} = a_z(L)^3 + b_z(L)^2 + c_zL + d_z$$

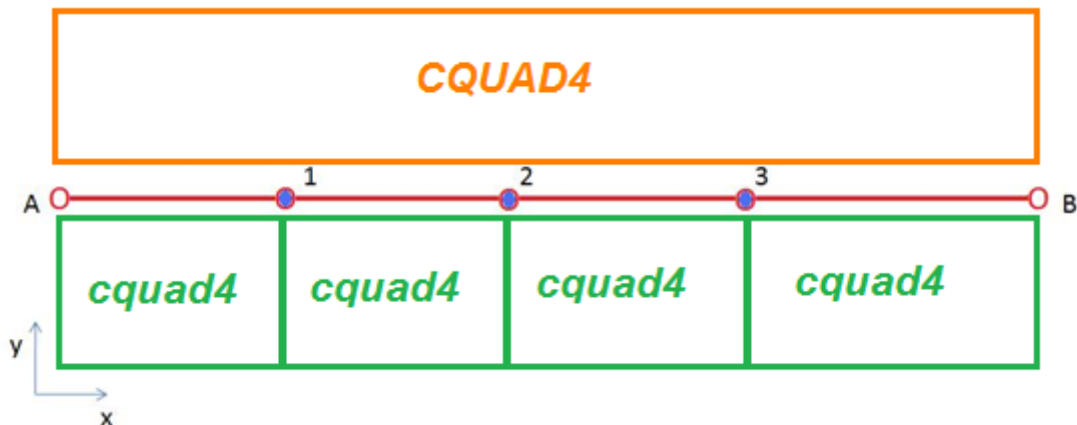
$$\theta_y^{node A} = c_z$$

$$\theta_y^{node B} = 3a_z(L)^2 + 2b_zL + c_z$$

Solving this system of equations the constants of the RSPLINE are found and the interpolation can be carried out for the calculation of the dependent nodes.

3. PROBLEMS ON STATIC ANALYSIS

The main problem of the RSPLINES on the static analysis is inherited from its transversal behavior. The RSPLINES commonly interpolate meshes of shell elements (CTRIA3 or CQUAD4).



The interpolation of the shell elements (CQUAD4/CTRIA3) is:

- Linear for the in-plane translations on the edges ('Tx' and 'Ty')
- Cubic for the out-of-plane translations on the edges ('Tz')
- Quadratic for the rotations in 'Ry' along the edges parallel to 'x'
- Quadratic for the rotations in 'Rx' along the edges parallel to 'y'
- Linear for the rotations in 'Rz' along the edges parallel to 'x'

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- Linear for the rotations in 'Ry' along the edges parallel to 'y'
- There is no out-of-plane rotation degree of freedom for these kind of elements (no 'Rz')

According to the behavior of the RSPLINE explained on the previous chapter:

- The interpolation is cubic for all the transversal displacements ('Ty' and 'Tz')
- The interpolation is linear for the longitudinal displacements ('Tx')
- The interpolation is quadratic for the transversal rotations ('Ry' and 'Rz')
- The interpolation is linear for the longitudinal rotation ('Rx')

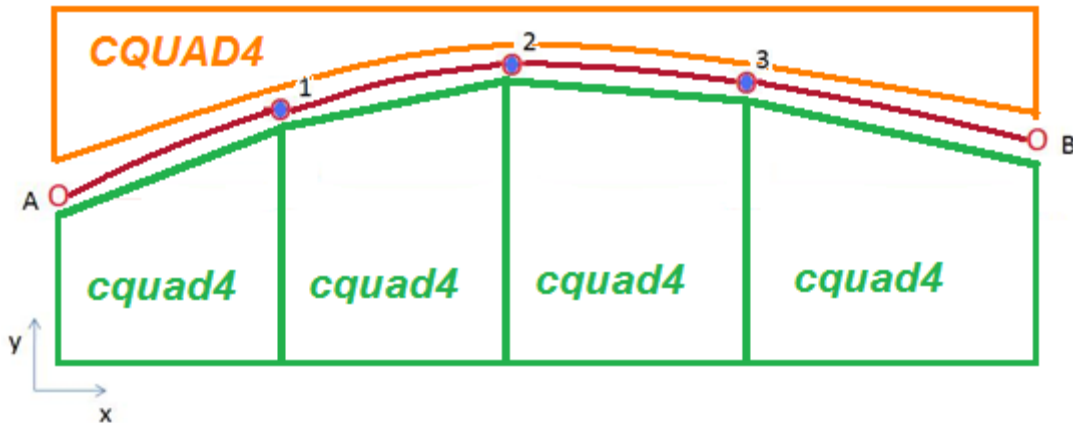
Hence, the displacements are interpolated as shown in the following table:

DISPLACEMENT	Is interpolated as	Should be interpolated as
Tx	Linear	Linear
Ty	Cubic	Linear
Tz	Cubic	Cubic
Rx on edges parallel to x	Linear	Linear
Rx on edges parallel to y	Not applicable	Quadratic
Ry on edges parallel to x	Quadratic	Quadratic
Ry on edges parallel to y	Not applicable	Linear
Rz	Quadratic	Not relevant

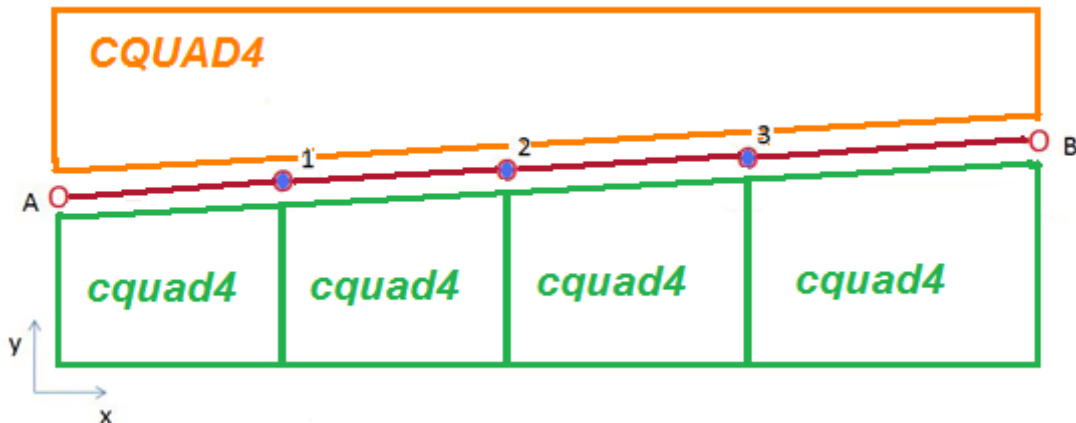
This leads to the conclusion that the error created by the RSPLINE interpolation is basically focused on the 'Ty' direction, meaning the translation on the direction perpendicular to the transition (to the RSPLINE) contained on the plane of the shell elements mesh.

For example, when enforced displacements coming from the coarse mesh are applied at the extremes of the RSPLINE (where the degrees of freedom are independent, points 'A' and 'B') the interpolation of the RSPLINE can lead to the following behavior (due, for example to the out of plane rotations, which are in reality spurious values):

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When the correct and expected behavior is like the following one:



The RSPLINES applied directly to interpolate meshes will create on the first behavior incorrect stresses on the 'y' direction on the central cquad4 of the fine mesh. These erroneous stresses can be positive or negative depending on the sign of the curvature incorrectly created by the interpolation carried out by the RSPLINE.

Note that this behavior is due to a modification of the model stiffness regarding the stiffness of a right modeled structure due to the use of RSPLINES in 'y' direction. As the RSPLINE imposes a cubic interpolation in 'Ty' it is enforcing the intermediated nodes to satisfy the cubic interpolation. This does not represent properly the behavior of the structure which is a linear interpolation.

The way to solve this incorrect behavior comprises two steps:

- Release the 'Ty' (transversal displacement on the plane of the meshes) on the dependent nodes of the RSPLINE. This way, the 'Ty' displacement will not be

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calculated and interpolated by using the cubic approach of the RSPLINE.

- In order to perform a correct transition of the meshes a set of MPC cards have to be created in order to interpolate the 'Ty' displacements linearly. These new MPC have to interpolate only the 'Ty' degree of freedom taking into account the distance to the independent nodes of the dependent node where the displacement is calculated

The RSPLINE defined as:

```
$111111122222223333333344444444555555556666666677777777
RSPLINE 1      1  2  123456 3  123456 4
           123456 5
```

Will be transformed in the following set of cards:

```
SOL 101
CEND
...
MPC = 2
...
BEGIN BULK
$111111122222223333333344444444555555556666666677777777
RSPLINE 1      1  2  13456 3  13456 4
           13456 5
MPC  2  2  2  -1.  1  2  .75
      5  2  .25
MPC  2  3  2  -1.  1  2  .5
      5  2  .5
MPC  2  4  2  -1.  1  2  .25
      5  2  .75
...
```

Different aspects must be highlighted for the RSPLINE card.

Firstly, due to RSPLINE is consider as a rigid element it must not be call through the Case Control section.

Nodes order in the RSPLINE card must be the same that the order in the modeled line.

Finally, when the set of cards which replace the RSPLINE are used, MPC = (SID) has to be added on the case control in order to activate them.

To implement this solution for the mesh interpolation on the FEM, it is necessary to have

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a proper coordinate system assigned as CD to the nodes that are interpolated. One of the principal directions of the CD of the nodes must be the one that has to be interpolated by means of MPC (the one perpendicular to the RSPLINE but in the plane of the shell mesh).

In case of having the CD coordinate systems already imposed in the RSPLINE nodes, RBE2 can be used to solve this problem. In each dependent node of the RSPLINE a coincident node should be added which will be joined with the previous one by means of a RBE2. Nodes belonging to the RSPLINE will have the CD coordinate system in order to release the DOF desired. Remaining nodes will have the CD coordinate system already imposed. To avoid problems with the RBE2 cards, the independent nodes of those must be the dependent nodes of the RSPLINE.

The NASTRAN cards corresponding to this implementation are presented below. These cards carry on with the previous example in which the RSPLINE nodes are identified with ID from 1 to 5. The new nodes with imposed CD coordinate systems are identified with ID from 11 to 15.

```

...
RBE2 111 1 123456 11
RBE2 112 2 123456 12
RBE2 113 3 123456 13
RBE2 114 4 123456 14
RBE2 115 5 123456 15
...

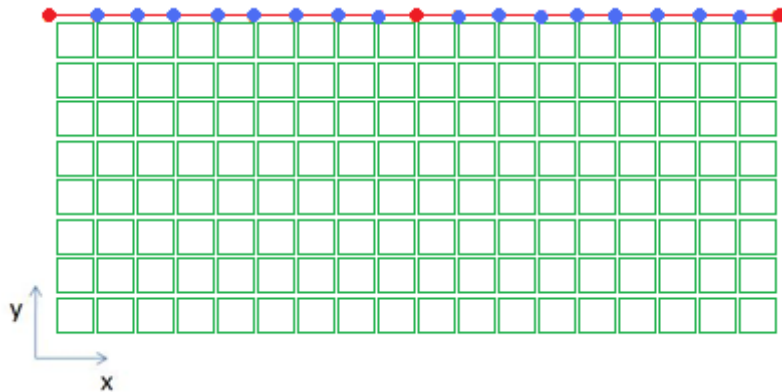
```

4. PROBLEMS ON BUCKLING ANALYSIS

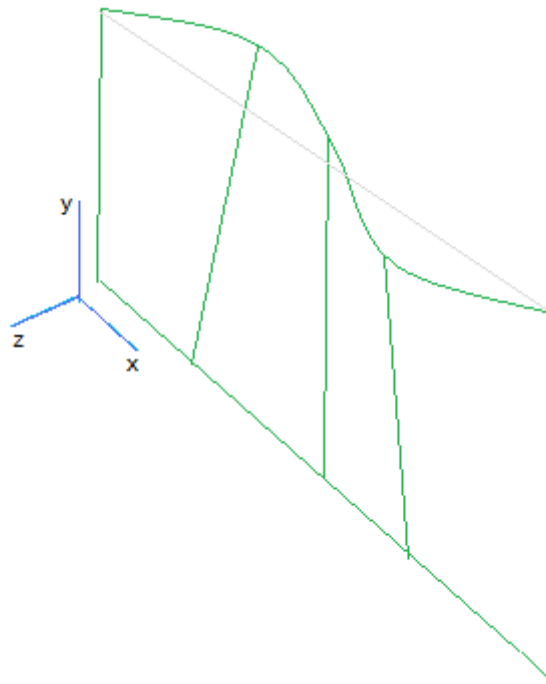
The RSPLINE can also lead to problems on the buckling analysis. The way to interpolate the displacements can lead to unexpected buckling shapes.

As an example, the following problem is presented:

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The upper edge of this fine mesh is interpolated by means of RSPLINE. For buckling the boundary condition desired in this case is simply supported (typical boundary condition applied because it is more conservative than clamping). The nodes of the fine mesh that are interpolated (central nodes of the RSPLINE) cannot be restrained by means of SPC as they are dependent nodes of the RSPLINE. Therefore, the only way to apply a simply supported condition on the upper edge for the buckling calculation is by constraining the translations on the independent nodes of the RSPLINE (the extreme nodes). This kind of restriction could lead to the following buckling shape:



Note that this boundary condition does not lead to the desired simply supported condition along the whole upper edge of the structure. The cubic interpolation of the 'Tz' displacements on the dependent nodes can potentially create a shape that fulfils the SPC



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requirement (as the independent nodes have null translations), and at the same time allowing the interpolated nodes to translate. This will not happen with a linear interpolation, as the interpolated value using a linear approach between null values is always null as well.

The way to avoid this problem on the buckling analysis is by using grounded springs (CELAS/CBUSH elements) with very high stiffness that connect the interpolated nodes with the duplicated nodes with SPC in 'Tz'. How to determine the stiffness for the spring elements is explained at the end of this document as well as the reason of using springs. The SPC will only be activated on the buckling subcase, not in the static subcase in order to avoid the distortion of the static analysis. This way the out-of-plane translation will be avoided on the buckling subcase and the buckling shape will be more in line with the desired behavior.

An example of the NASTRAN file is presented below.

```

SOL 105
CEND
...
MPC = 2
...
SUBCASE 1
SPC = 2
LOAD = ...
...
SUBCASE 2
SPC = 1
METHOD = 1
...
BEGIN BULK
$111111122222223333333344444444555555556666666677777777
RSPLINE 1      1  2  13456 3  13456 4
           13456 5  13456 6
MPC  2  2  2  -1.  1  2  .8
           6  2  .2
MPC  2  3  2  -1.  1  2  .6
           6  2  .4
MPC  2  4  2  -1.  1  2  .4
           6  2  .6
MPC  2  5  2  -1.  1  2  .2
           6  2  .8
...

```


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```

CELAS2 212 1.+9 2 3 112 3
CELAS2 213 1.+9 3 3 113 3
CELAS2 214 1.+9 4 3 114 3
CELAS2 215 1.+9 5 3 115 3
...
SPC1 1 123456 112 113 114 115
SPC1 2 12456 112 113 114 115
...

```

This example has the same geometry as the previous NASTRAN example. Nodes from 112 to 115 are the duplicated nodes which provide stiffness in 'Tz' on the buckling subcase by means of SPC.

5. CONSTRAIN MPCs OR R-type ELEMENT NODES

Three important rules that must be fulfilled when using MPCs or R-type elements.

- Dependent DOFs of this kind of elements cannot be constrained.
- A DOF cannot be dependent on two different MPC.
- The R-type elements must be statically determined.

Due to these rules, if it is desired to constraint a dependent DOF, the following solution can be applied. As a real constrain such as SPC cannot be use, it must be modeled as a high stiffness element. The simplest way to do that is using springs with high stiffness. In order to avoid spurious forces and other kind of problems, springs should have null length. Therefore duplicated nodes will exist in the dependent nodes to be constrained. The duplicated non-dependent nodes should be grounded, that is with all DOFs constrained. These nodes will be joined with the previous mentioned high stiffness springs, only in the desired directions.

With the purpose of impose the highest stiffness for the springs, and therefore simulate as correctly as possible the constraints the following process can be carried out. First of all, a really high stiffness is imposed, for example $1e+14$. With the previous stiffness the NASTRAN code is run obtaining a Fatal error in the output. This fatal error is a result of exceeding the MAXRATIO parameter which implies singularities in the stiffness matrix. The default value of this parameter is $1e+7$. Once the value which has been exceeded the MAXRATIO is obtained, the new stiffness value can be computed. The stiffness of the springs must be decreased to obtain the maximum allowed MAXRATIO. So, it could be necessary to perform some loops until the highest stiffness (with no Fatal error) is achieved.